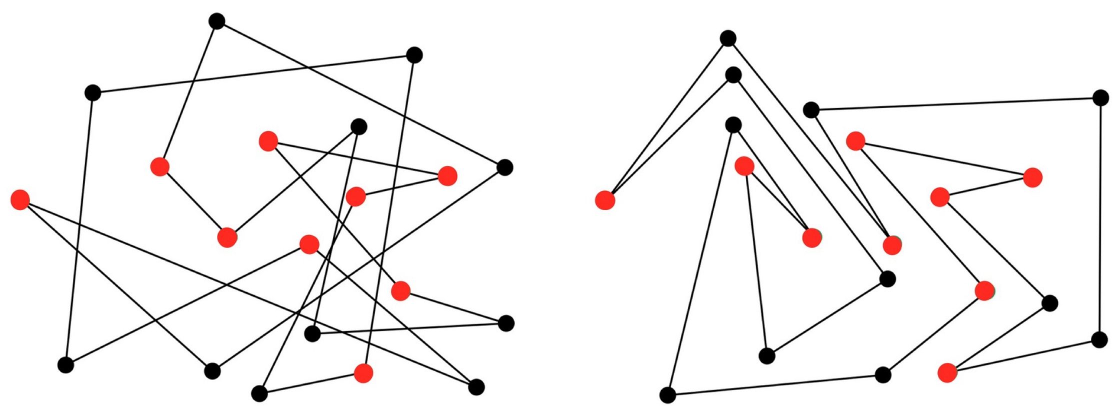
Problem 2:

Q: [Pach, Tardos] 2002

Can every geometric planar graph be untangled while keeping $\geq n^{\varepsilon}$ vertices fixed?



Problem 2:

Q: [Pach, Tardos] 2002

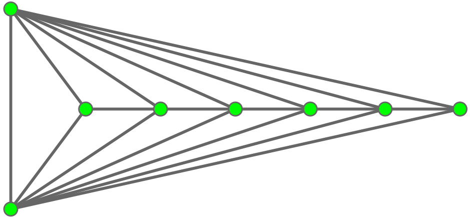
Can every geometric planar graph be untangled while keeping $\geq n^{\varepsilon}$ vertices fixed?

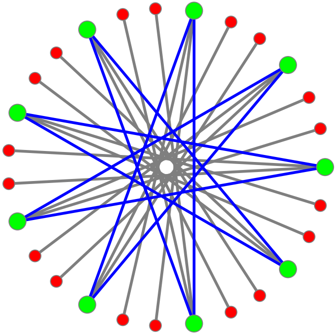
→ SL in BB

Geometric Graphs

geometric (planar) graph:

- vertices: points in the plane
- edges: straight-line segments

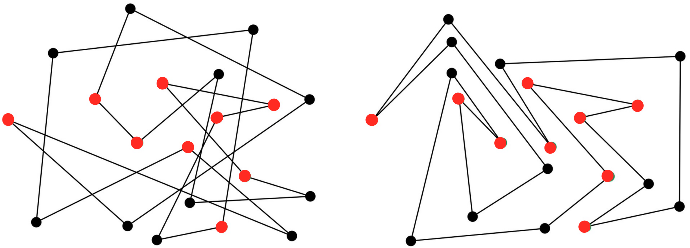




problem statement: Untangling Planar Graphs

given: a geometric planar graph **G**

task: move as few vertices as possible, such that the resulting geometric planar graph is crossing-free; that is, untangle **G**.



problem statement: Untangling Planar Graphs

given: a geometric planar graph **G**

task: move as few vertices as possible, such that the resulting geometric planar graph is crossing-free; that is, untangle **G**.

By Fáry / Wagner's Theorem:
Every geometric planar graph can be *untangled*.

Planarity game

question:

Can planar graphs be untangled while keeping $\mathbf{n}^{\varepsilon}$ vertices fixed?

geometric

[Pach and Tardos:2002, DCG]

question:

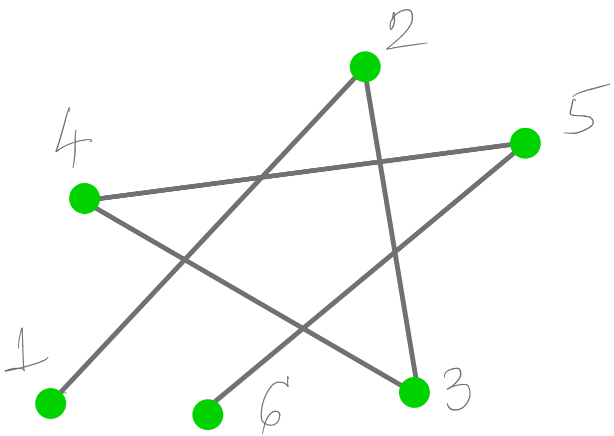
Can planar graphs be untangled while keeping $\mathbf{n}^{\varepsilon}$ vertices fixed?

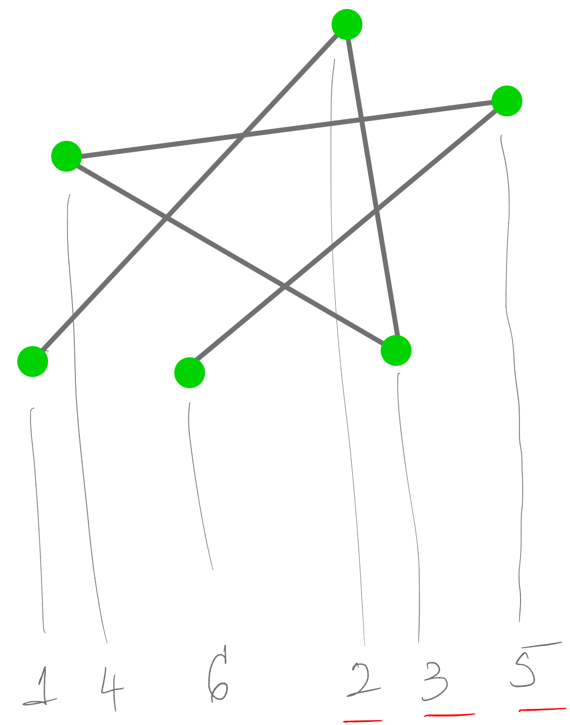
geometric

[Pach and Tardos:2002, DCG]

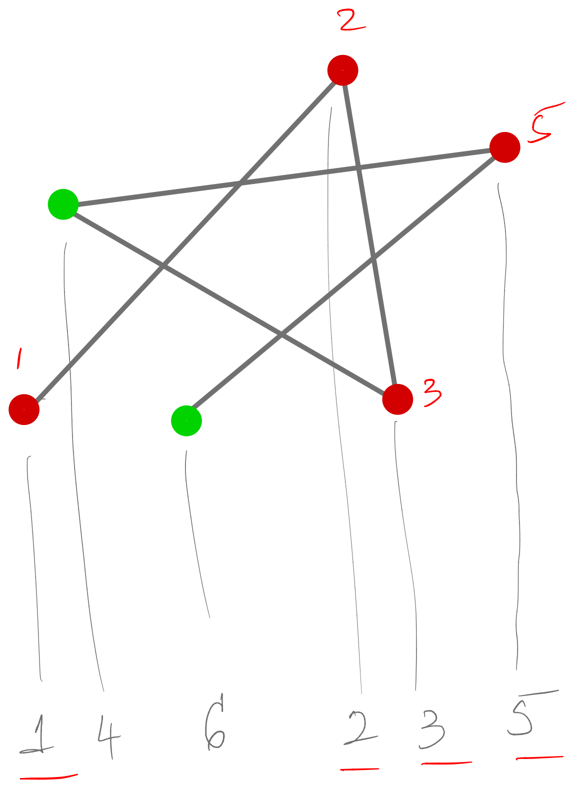
Can you do it for
$\forall$ geom. cycle/path?

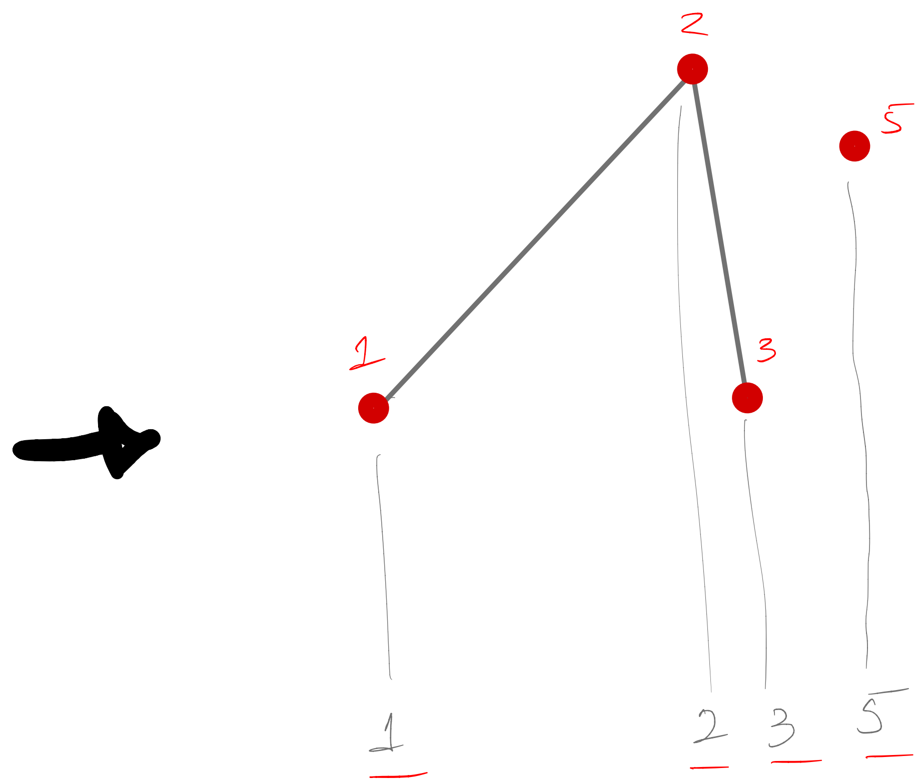
Untangling a path



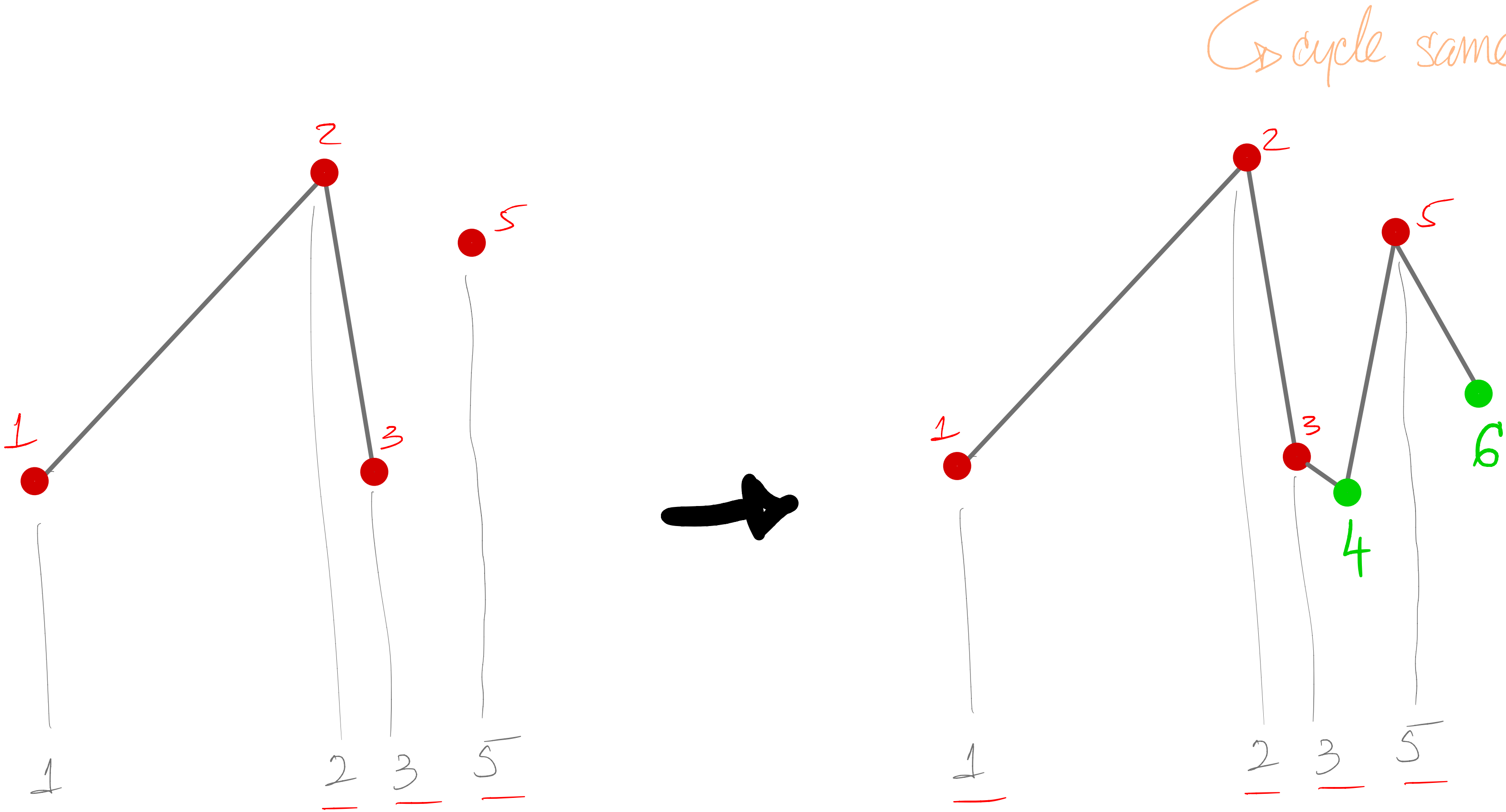


Untangling a path



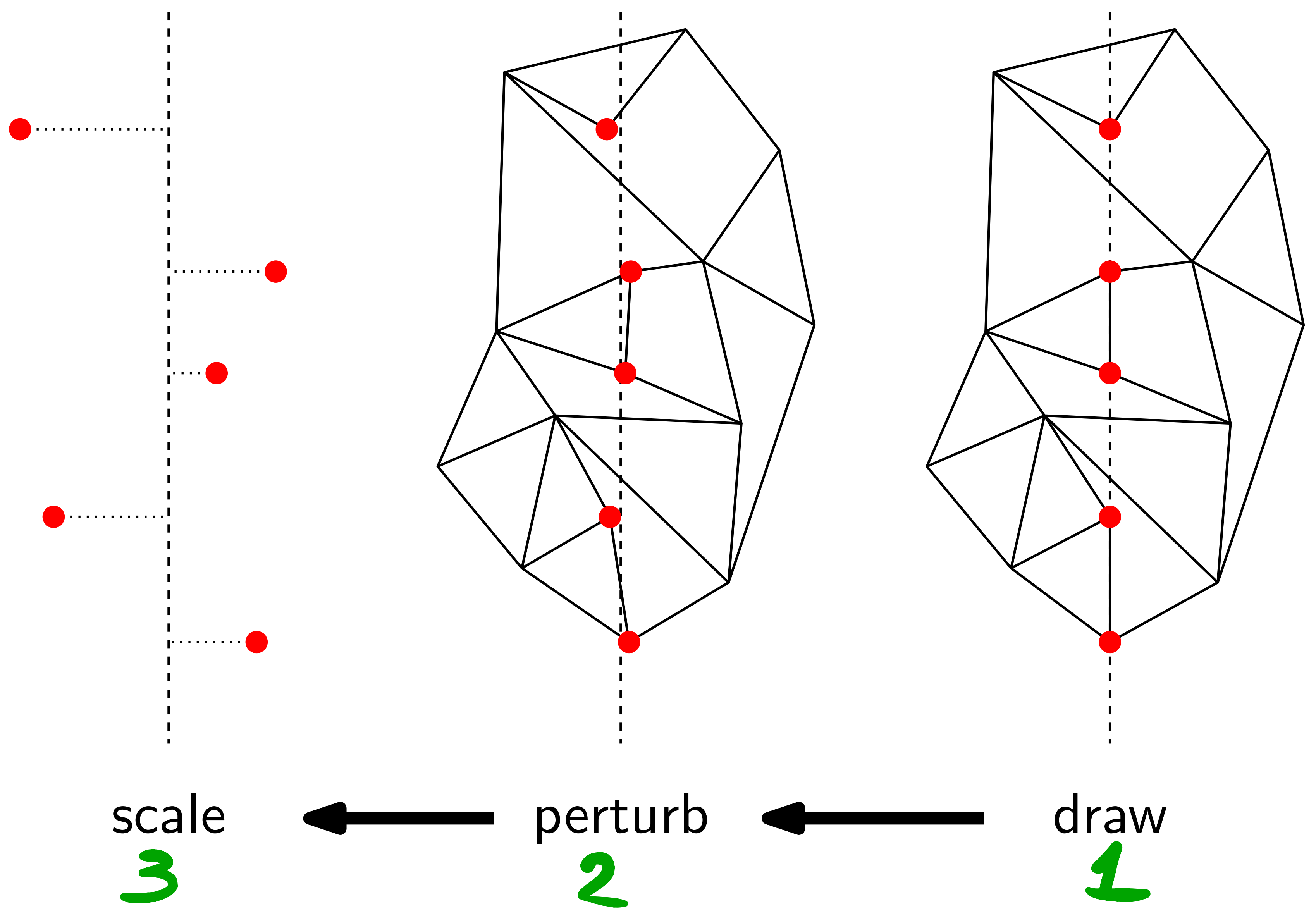


Untangling a path



günter

A FACT:



Collinear Sets

Def: $S \subseteq V(G)$ is a collinear set of G if G has a crossing-free SL drawing with S on a line.

Collinear Sets

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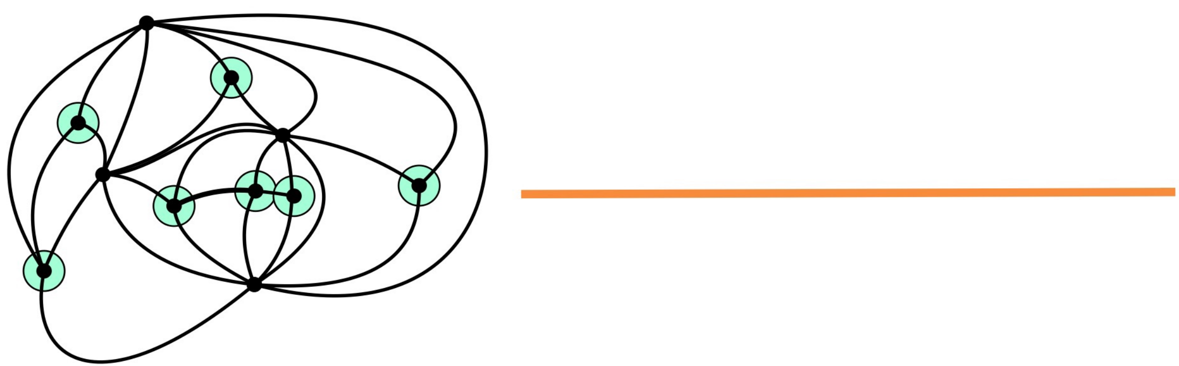

Collinear Sets

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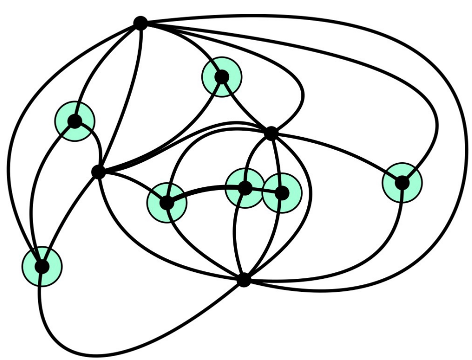
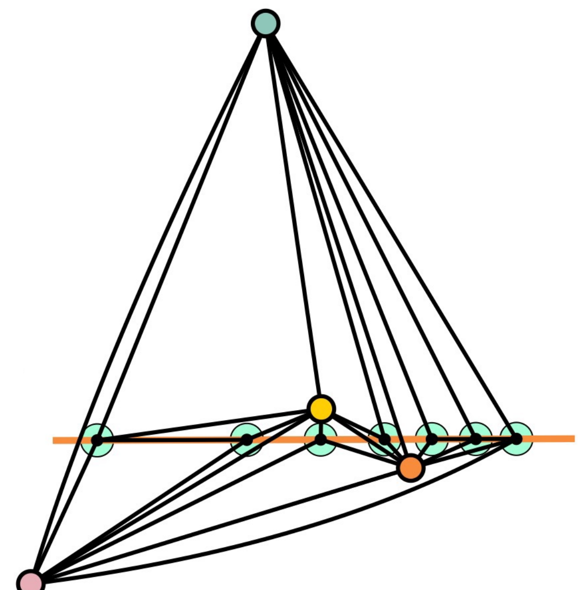
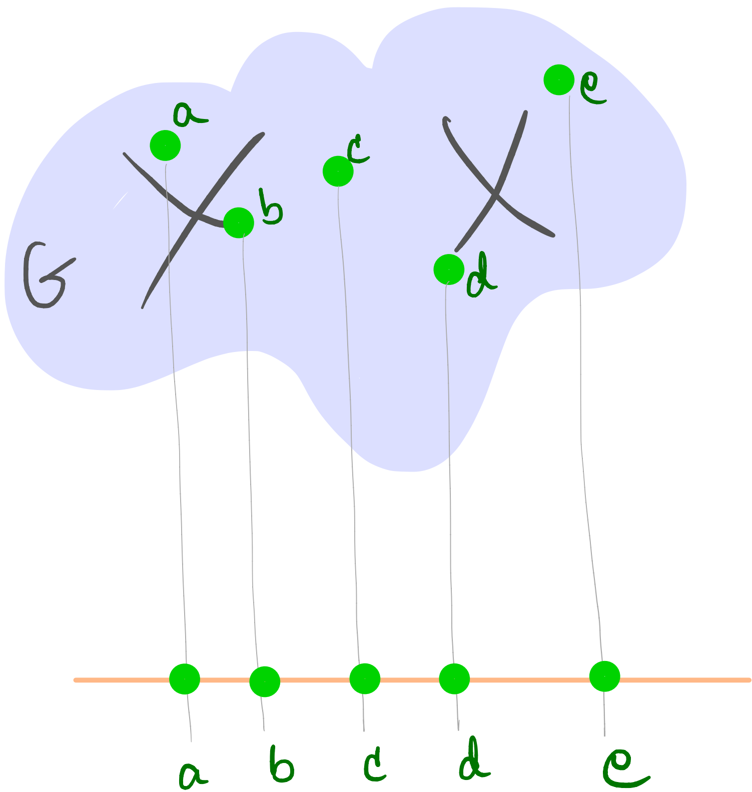




Collinear sets - do they help?



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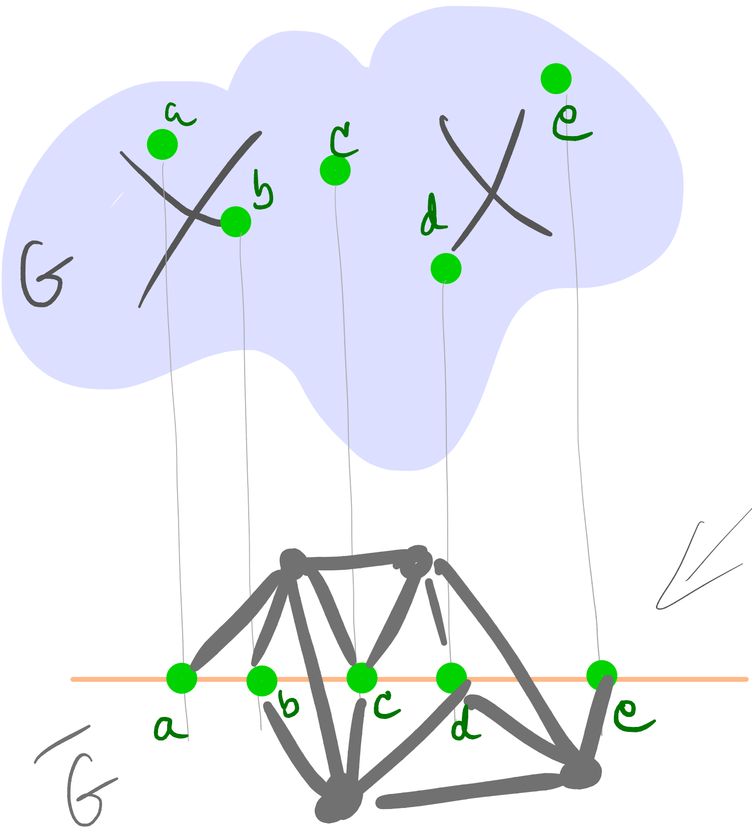
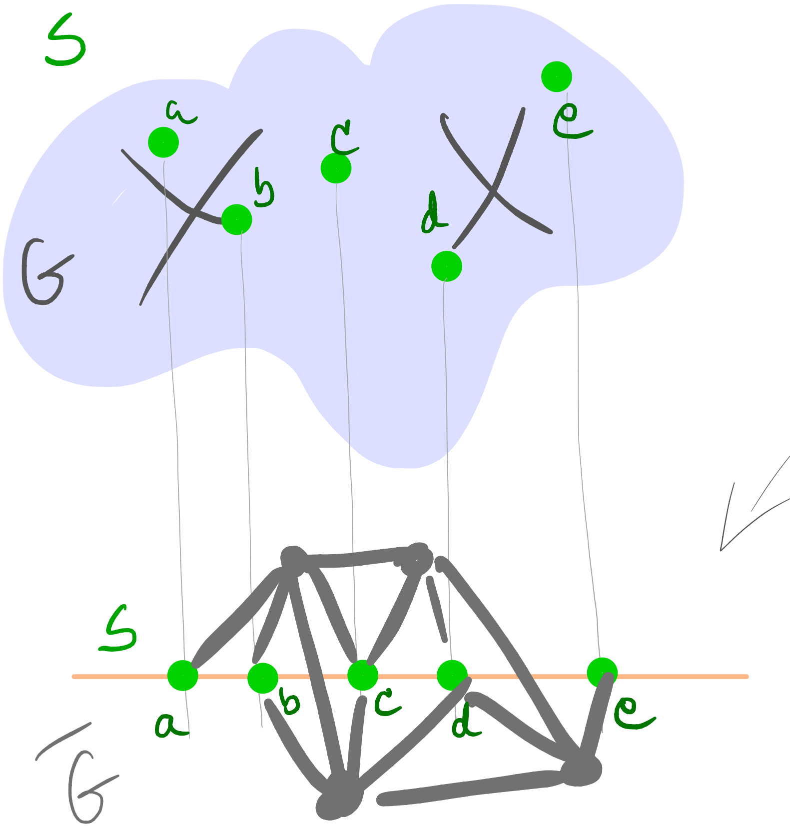


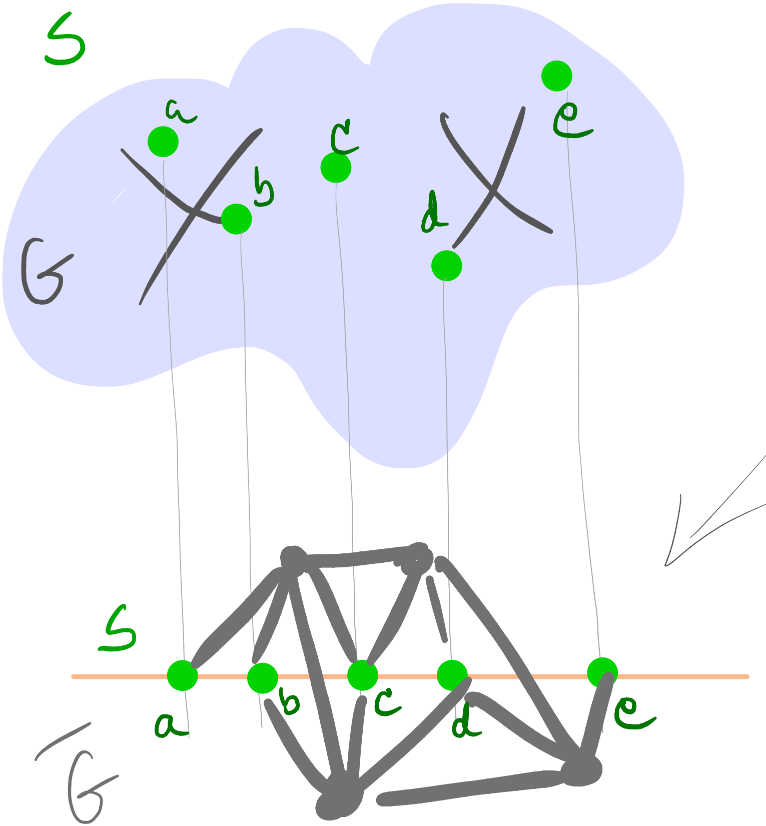
If this drawing ∃

Collinear sets - do they help?



If this drawing ∃ then G can be untangled while keeping S fixed by the fact.

Collinear sets - do they help?

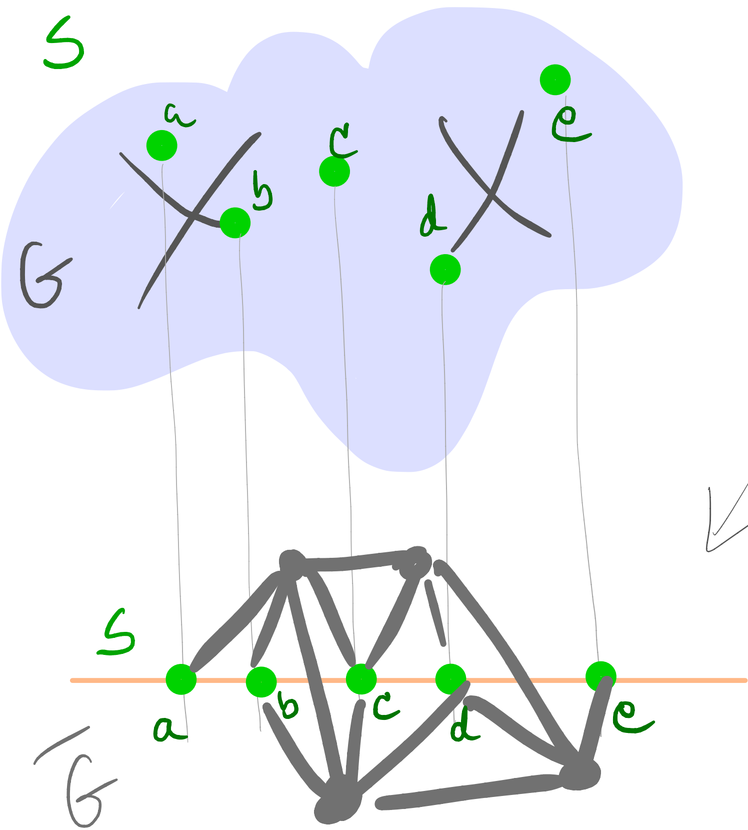


If this drawing ∃

then G can be untangled while keeping S fixed by the fact.

then collinear set S usefull

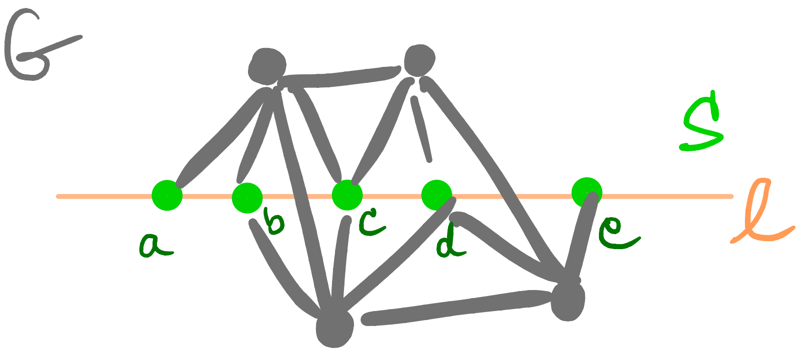
Collinear Sets - do they help?



If this drawing ∃

BIG IF

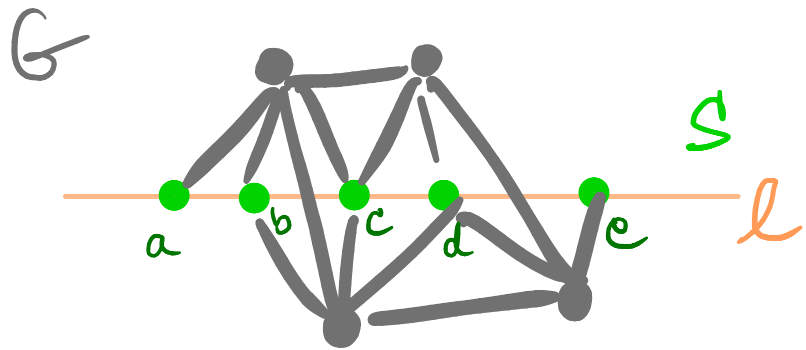
Free Collinear Sets

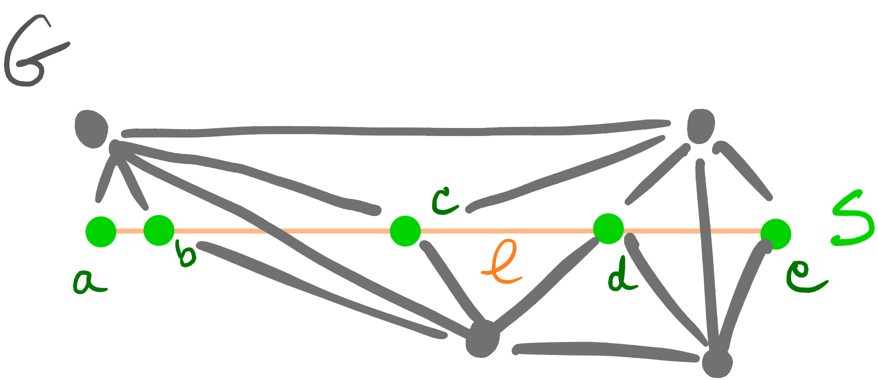


Def: $S \subseteq V(G)$ is **free col. set** if vertices of S can "freely move" on ℓ and still G has CFSTL.

(while keeping the ordering)

Free Collinear Sets

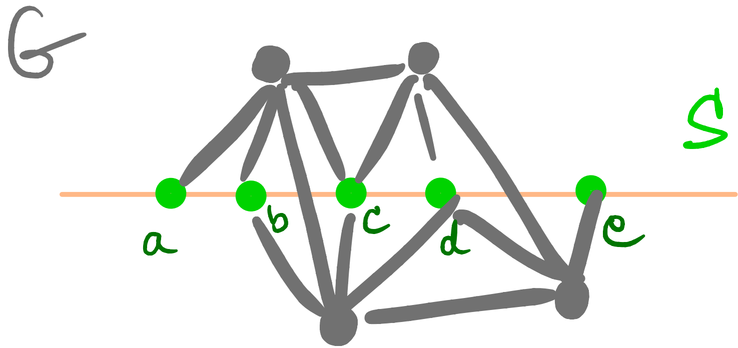




Def: $S \subseteq V(G)$ is free col. set if vertices of S can can "freely move" on ℓ (while keeping the ordering) and still G has CFSTL.

Free Collinear Sets

why? does it help untangle?



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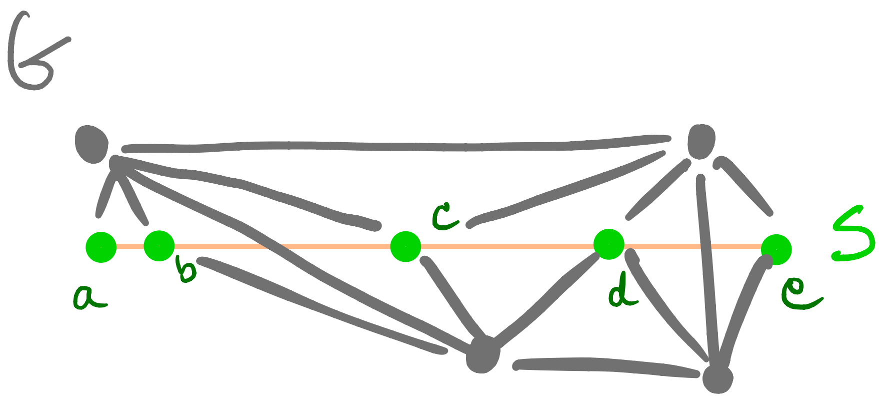
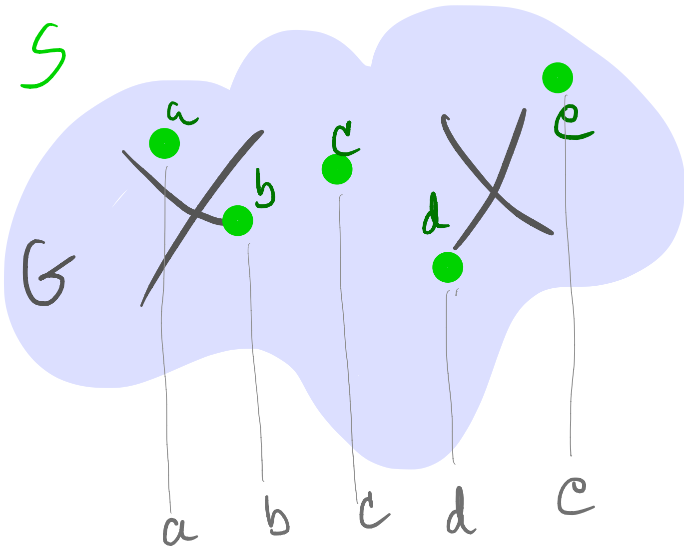


Why Free Collinear Sets?



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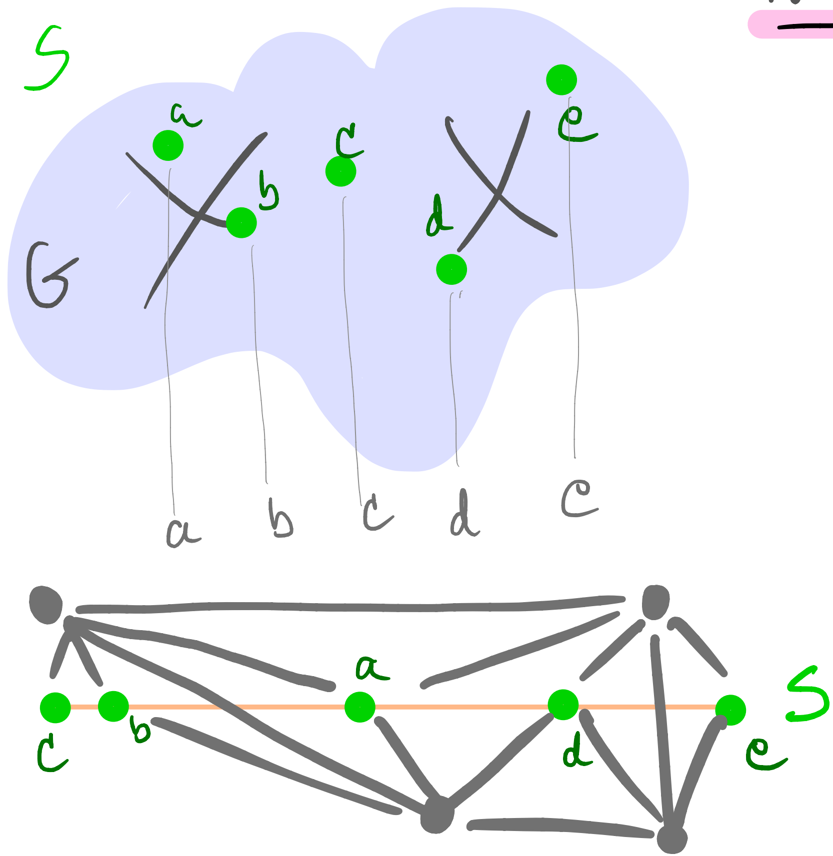

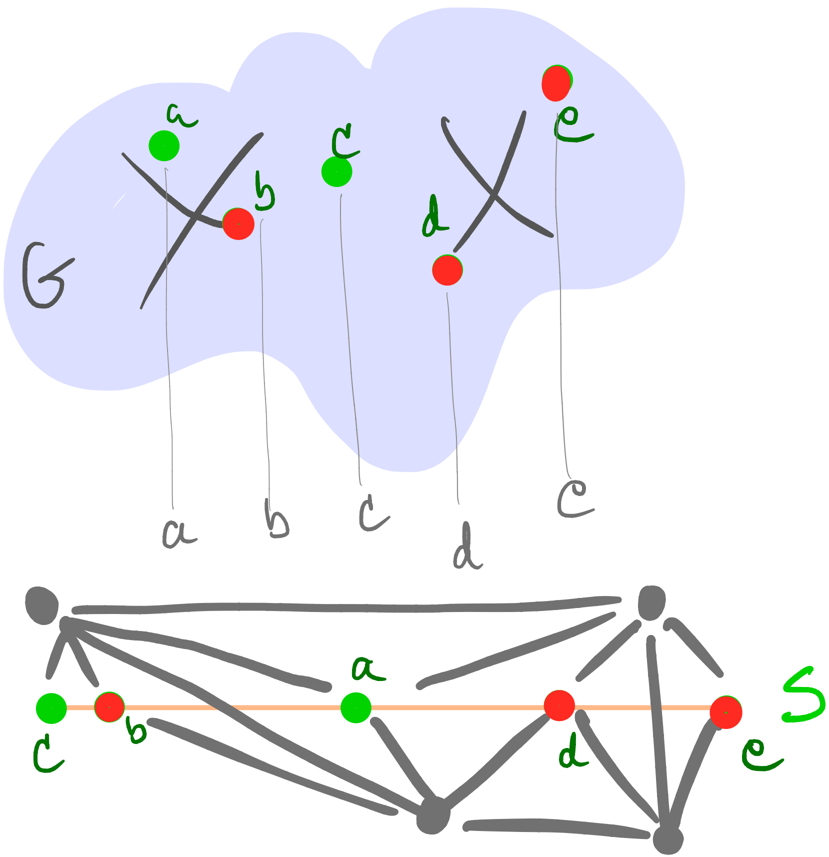

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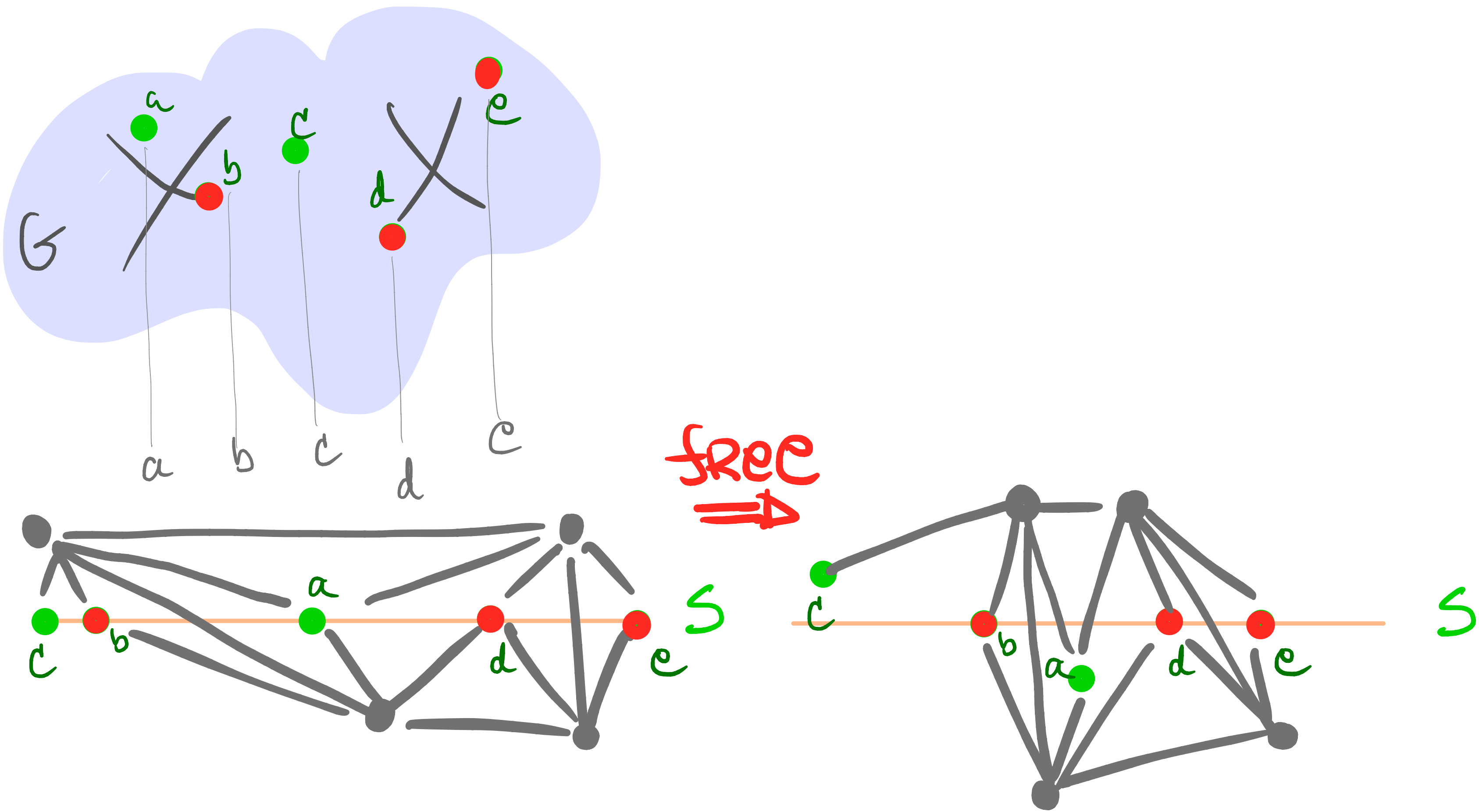
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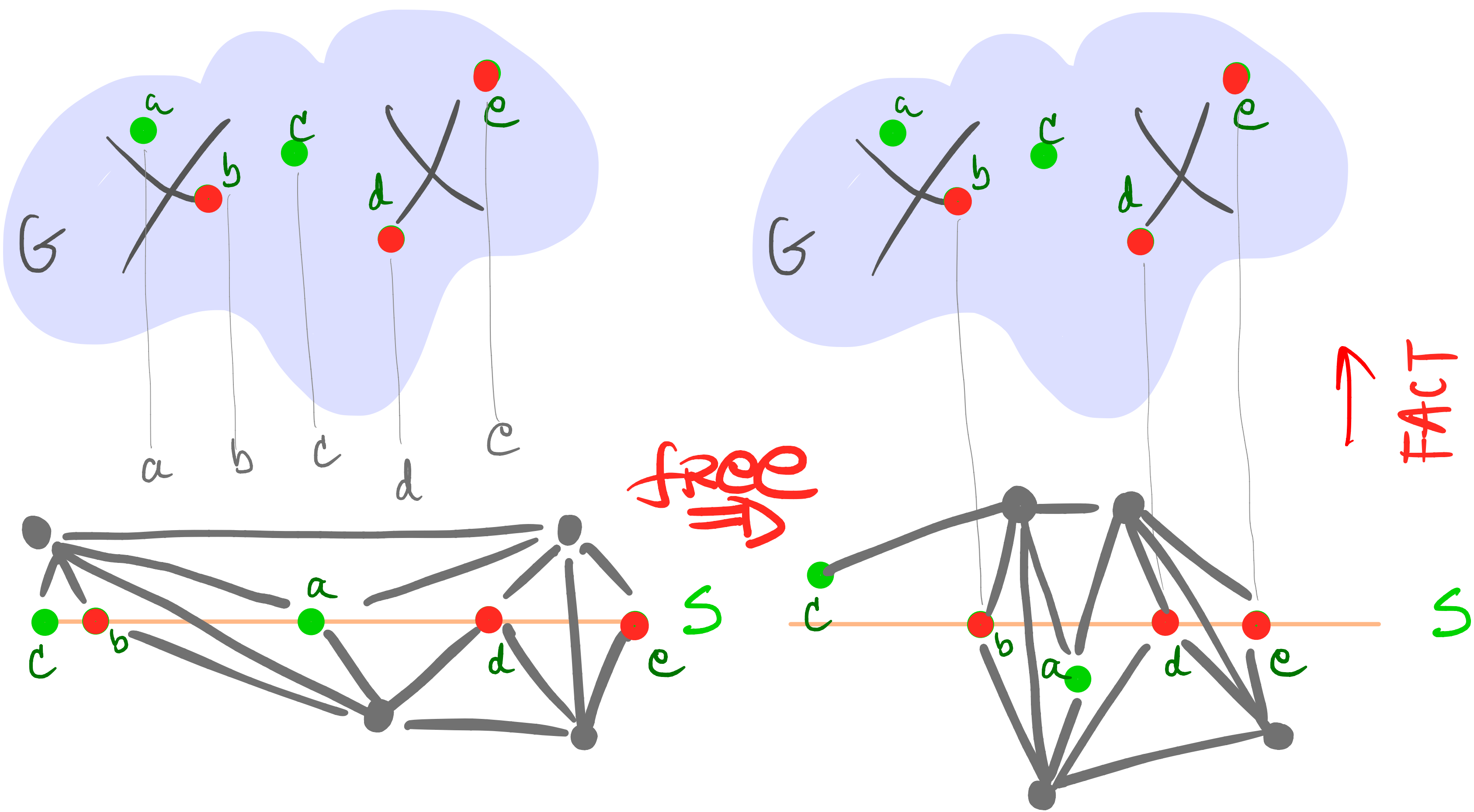
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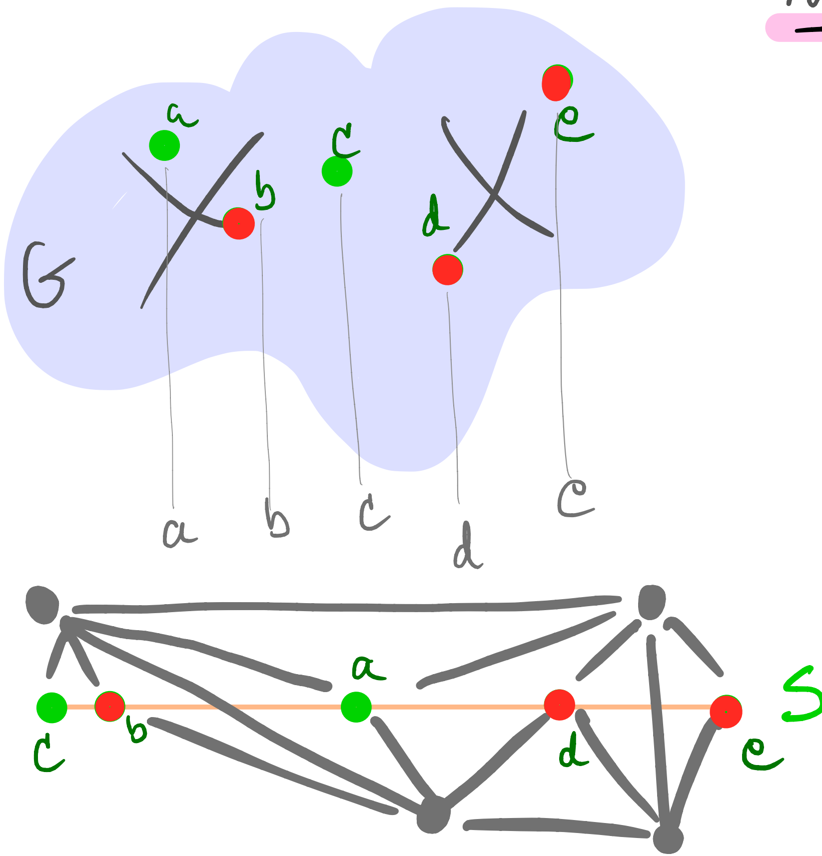
Why Free Collinear Sets?



Can fix :

$$\sqrt{|\mathrm{FreeColSet}(G)|}$$

$\forall$ geometric planar graph G

$$\text{fix}(G) \geq \sqrt{|\text{FREE COL. SET of } G|}$$

Graph classes with large free col. sets?

large → $\Omega(n^{\varepsilon})$, $0 < \varepsilon \leq 1$

- path?
- cycle?
- outerplanar?
- planar?

Graph classes with large free col. sets?

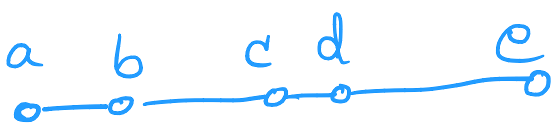
→ $\Omega(n^{\varepsilon})$, $0 < \varepsilon \leq 1$

FREE

- path ✓ n
- cycle ?
- outerplanar ?
- planar ?

untangled

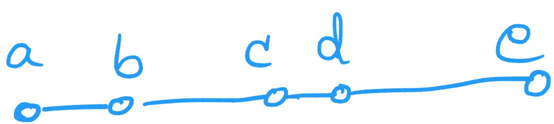
$\Omega(n)$

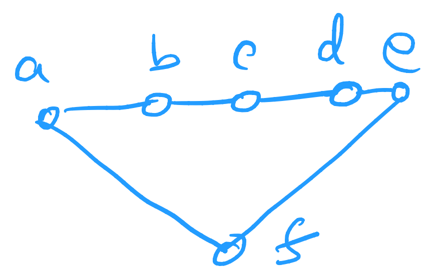


Graph classes with large free col. sets?

→ $\Omega(n^{\varepsilon})$, $0 < \varepsilon \leq 1$

	FREE	UNTANGLE
– path ✓	n	$\sqrt{n}$
– cycle ✓	$n-1$	$\sqrt{n-1}$
– outerplanar ?		
– planar ?		





previous work: subclasses of planar graphs

UNTANGLING G

graph class $\mathcal{G}$	lower bound	upper bound
cycles	$\Omega(n^{2/3})$ [Cibulka'08]	$\mathcal{O}(n \log n)^{2/3}$ [Pach&Tardos'08]
trees	$\sqrt{n/2}$	? [Spillner & Wolff]
outerplanar	$\Omega(\sqrt{n})$ [Spillner & Wolff]	$\mathcal{O}(\sqrt{n})$ [Goaoc *et al.*'07]

previous work *untangling*

for $\mathbf{G}$ planar

- $\text{fix}(\mathbf{G}) \geq \mathbf{3}$ [Goaoc *et al*, GD 2007]
- $\text{fix}(\mathbf{G}) \geq \mathbf{c}\sqrt{\mathbf{\log n / \log\log n}}$ [Spillner and Wolff, 2007]

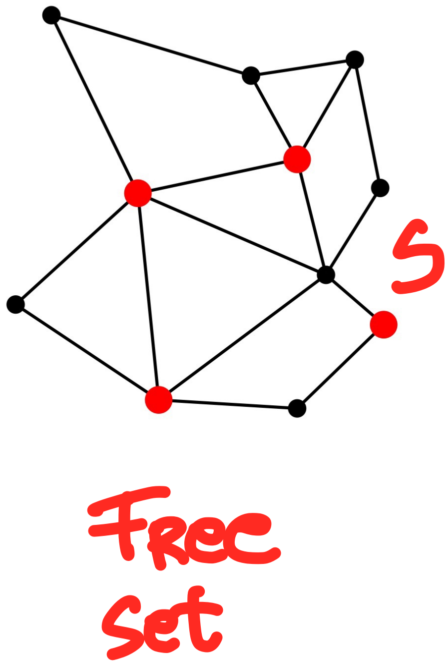
Graph classes with large free col. sets?

→ $\Omega(n^{\varepsilon})$

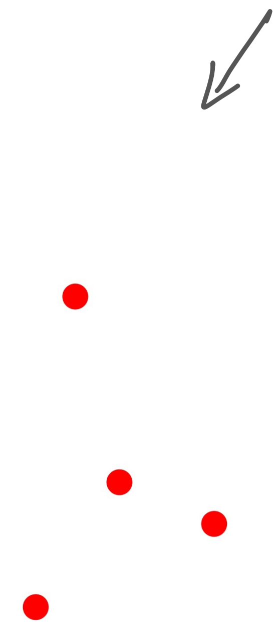
FREE COLLINEAR SETS SIZE:

- path ✓ n
- cycle ✓ $n-1$
- outerplanar ?
- planar ?

More reasons to care

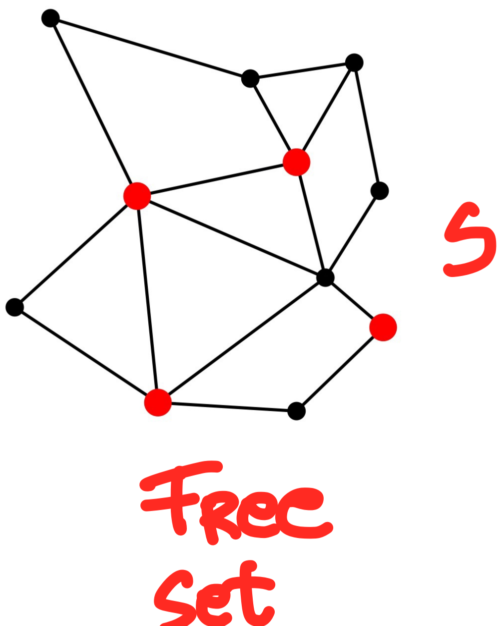


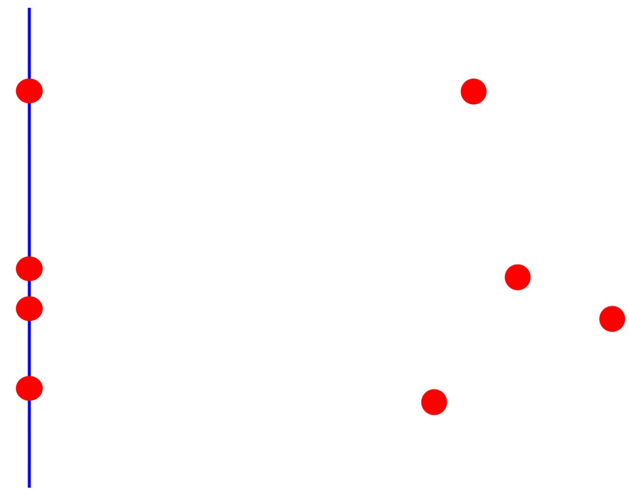
S free



More reasons to care

S is free collinear $\Rightarrow$ S' is free





Graph classes with large free col. sets?

large → $\Omega(n^{\varepsilon})$

~~FREE~~ COLLINEAR SETS SIZE:

→ easier?

- path ✓ n
- cycle ✓ $n-1$
- outerplanar ?
- planar ?

PLANAR GRAPHS

- free collinear sets WANT

  How can we find them?

PLANAR GRAPHS

- free collinear sets WANT

  How can we find them?

- collinear set: Are they easier to find?

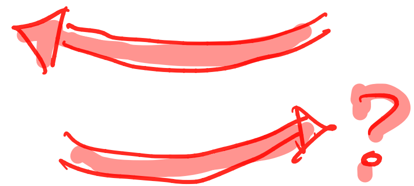
RELATIONSHIP

- free collinear sets WANT
  How can we find them?
- collinear set: Are they easier to find?

(1) • what is their relationship, if any?

RELATIONSHIP

collinear sets and free col. sets?



[Ravsky & Verbitsky 2006]

1. How far or close are parameters $\tilde{v}(G)$ and $\bar{v}(G)$? It seems that a priori we even cannot exclude equality. To clarify this question, it would be helpful to (dis)prove that every collinear set in any straight line drawing is free.

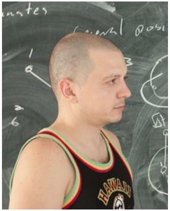
RELATIONSHIP

last

Th [D., Frati, Gonçalves, Morin, Rote] 2018

Every collinear set is free (collinear) set.

Free Sets in Planar Graphs


RELATIONSHIP

free collinear sets WANT

How can we find them?

collinear set:

RELATIONSHIP

free collinear sets WANT

How can we find them?

collinear set: WANT

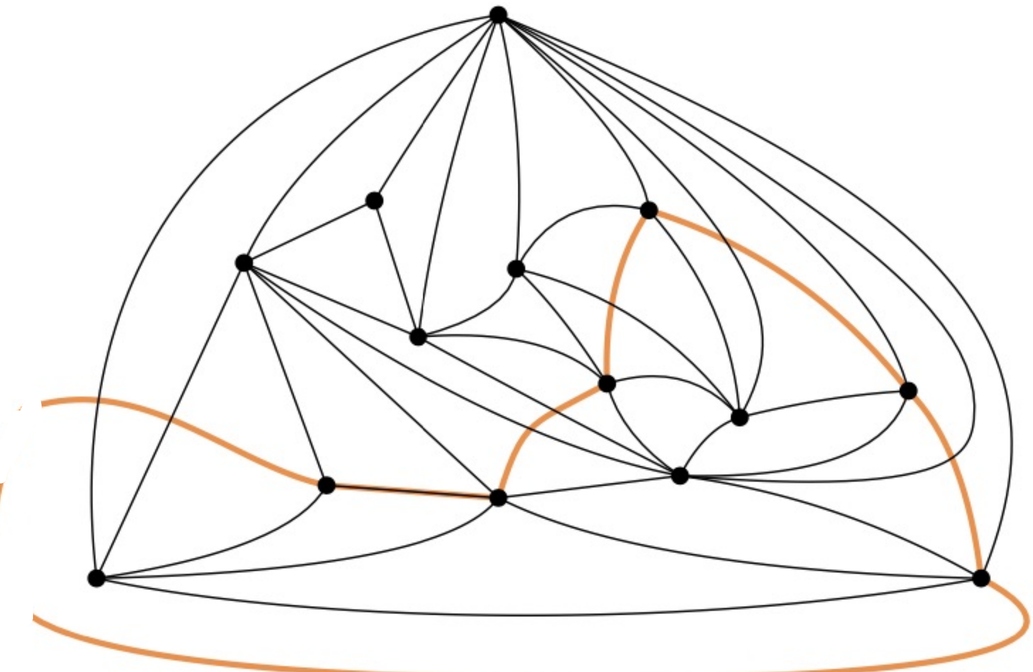
by finding these

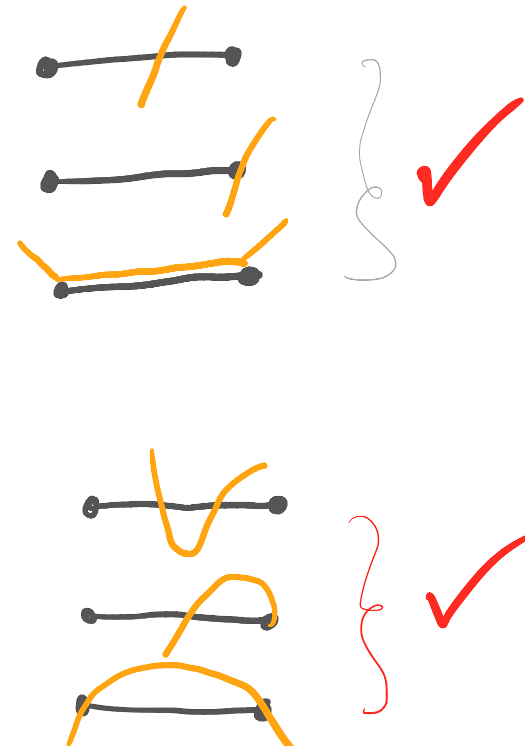
FINDING COLINEAR SETS

From geometry to topology

FINDING COLINEAR SETS

From geometry to topology



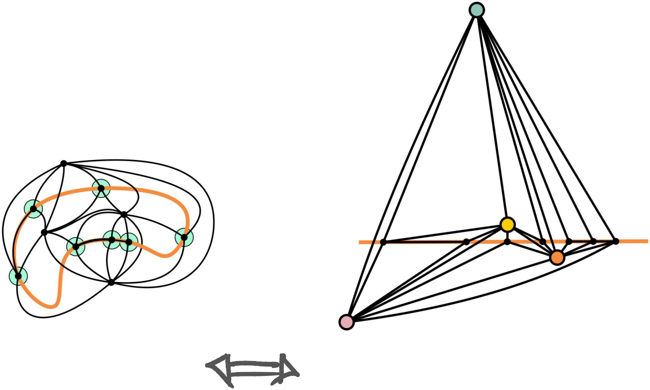


PROPER GOOD CURVE

Proper Good Curves and Collinear Sets

Theorem (Da Lozzo, Dujmović, Frati, Mchedlidze, Roselli (2018))

S is a collinear set iff some proper good curve contains S.



Proper Good Curves and Collinear Sets

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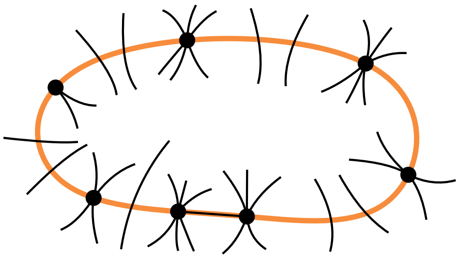
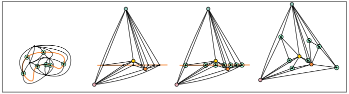
Proof sketch:





RELATIONSHIP

free collinear sets: WANT

collinear set: WANT

How do we find it?

RELATIONSHIP

free collinear sets: WANT

collinear set: WANT

proper good curve: WANT

↳ LOOK for this

PROPER GOOD CURVES

what is the largest ↑ that # planar G has?

PROPER GOOD CURVES

what is the largest ↗ that ∀ planar G has?

not linear: [Ravsky & Verbitsky 2007, 2011]

There exists n-vertex planar graphs whose ~~largest free set~~ proper good curve has size $O(n^{\log_{23} 22}) \subseteq O(n^{0.9859})$

PROPER GOOD CURVES

what is the largest ↑ that ∀ planar G has?

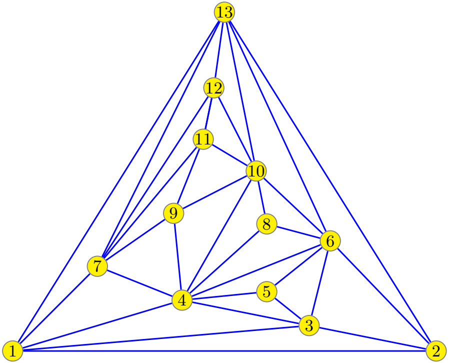
not linear [Pavsky & Verbitsky 2007, 2011]

at least $\Omega(\sqrt{n})$

[Bose, D., Hurtado, Langerman, Morin, Wood, 2007]

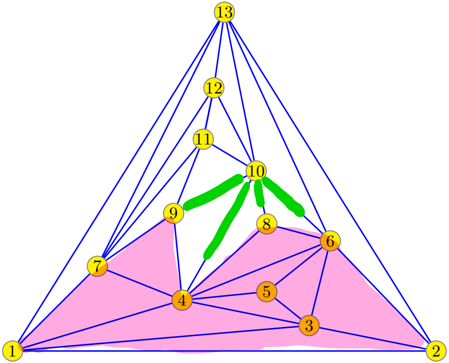
⟶ solves untangling question with $\text{fix}(G) \geq \Omega(n^{1/4})$

Frame



canonical ordering of $\mathcal{G}$

Frame



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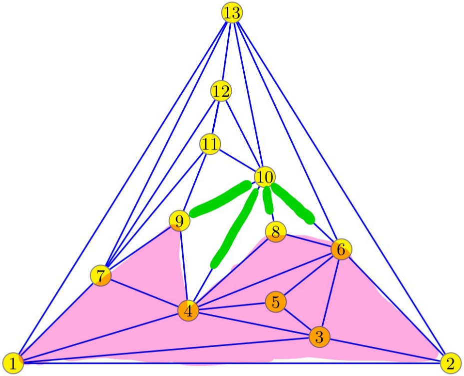
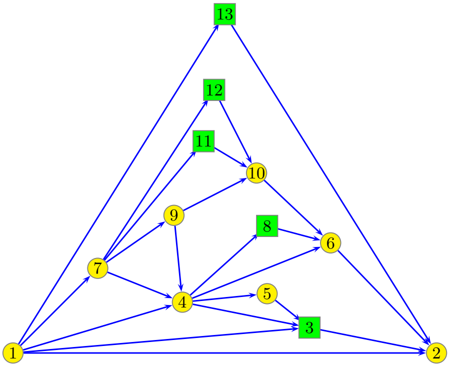
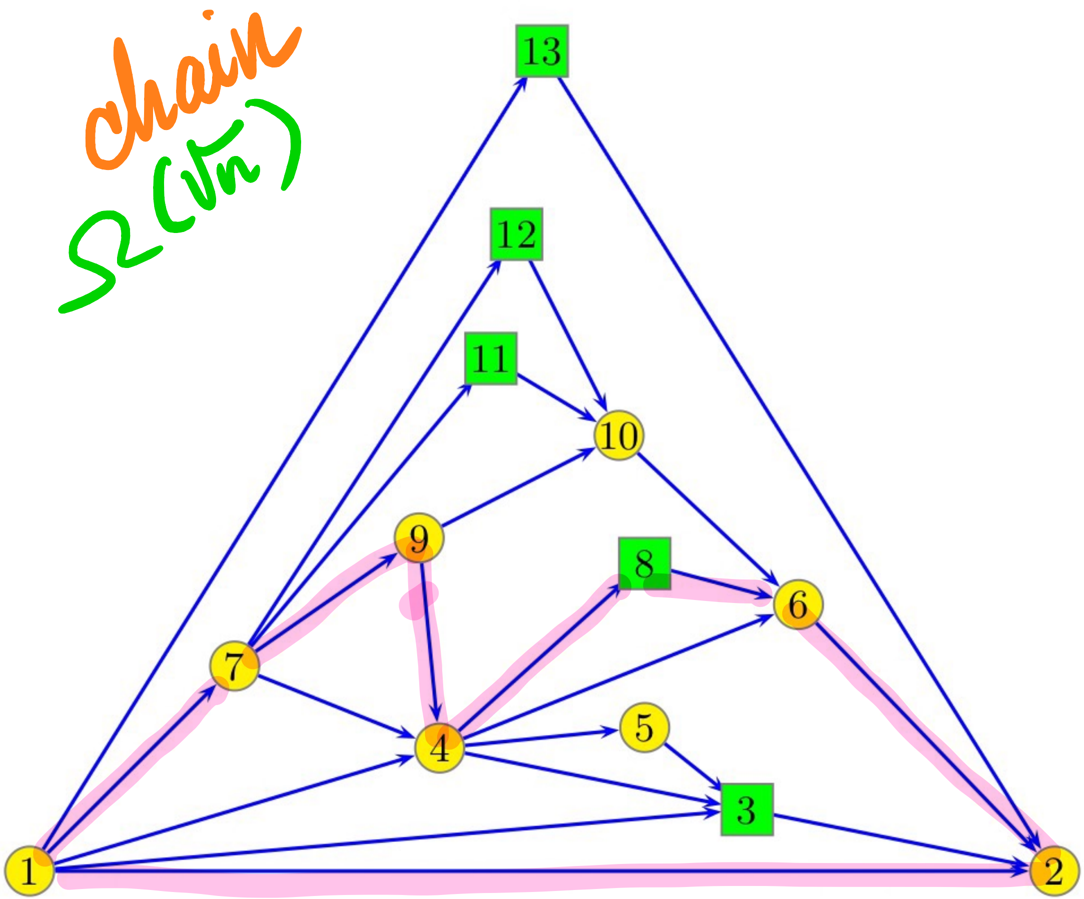


canonical ordering of $\mathcal{G}$



frame $\mathcal{F}$


chain
Ω(√n)
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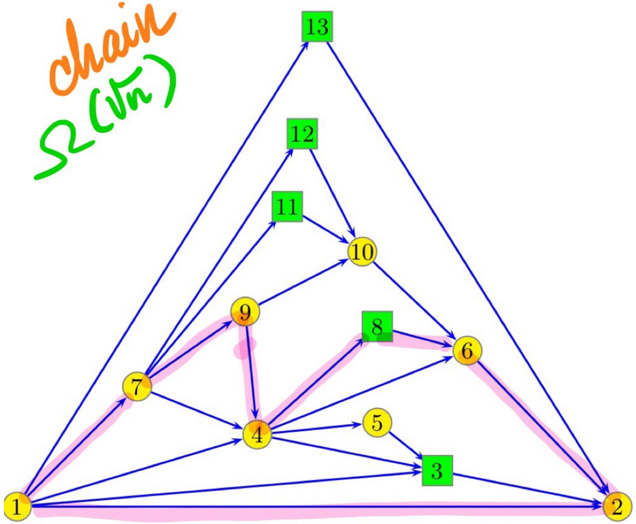
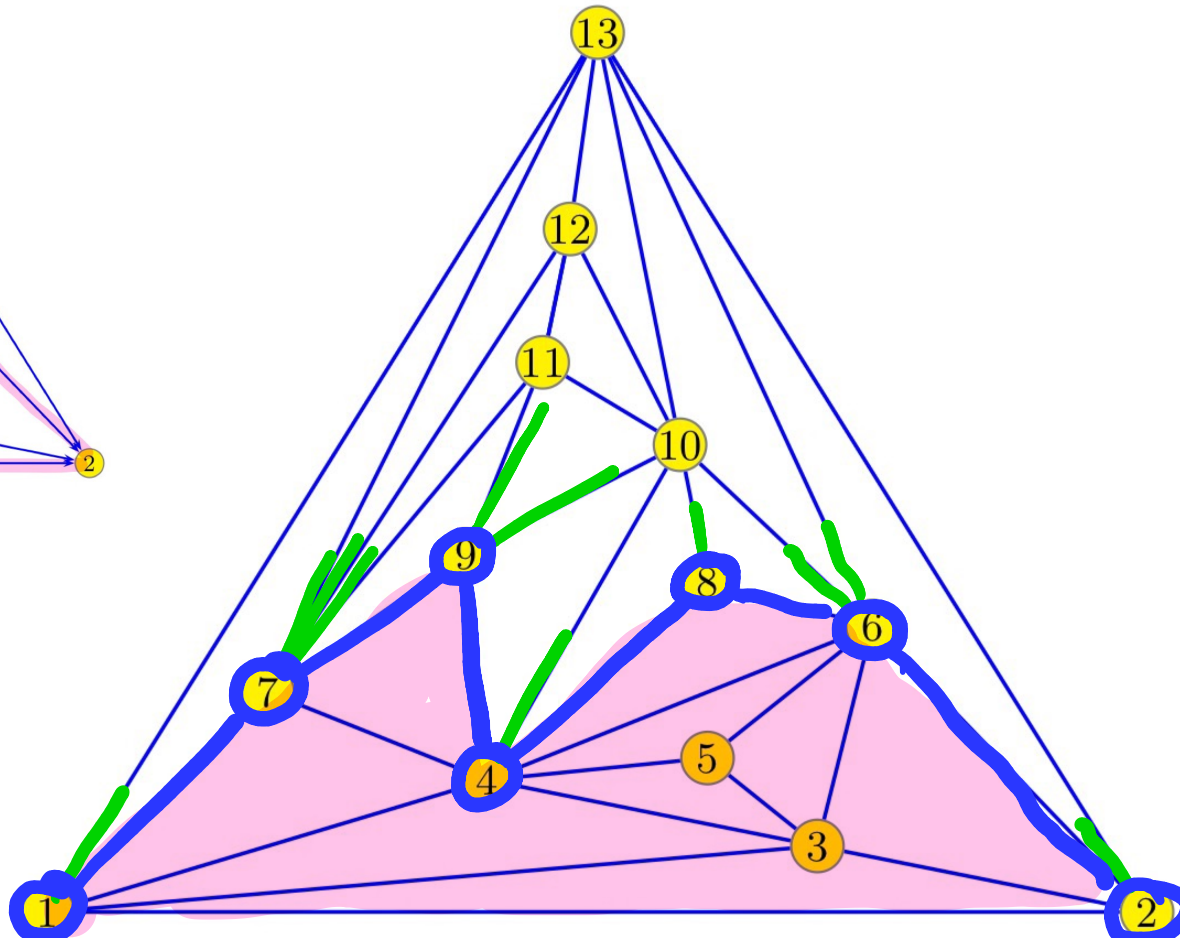
chain

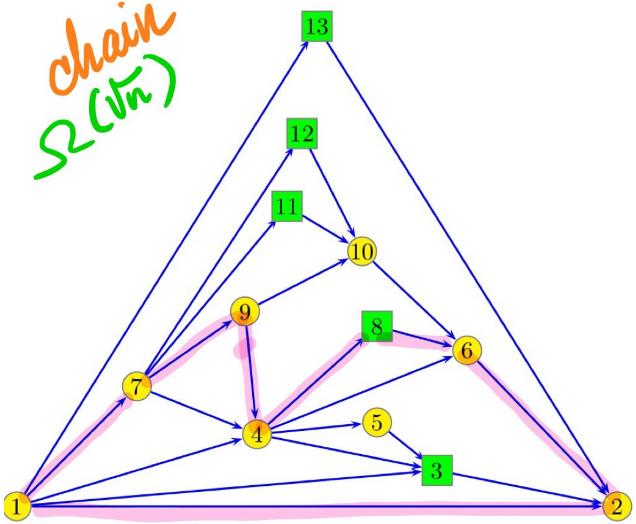
$\Omega(\sqrt{n})$

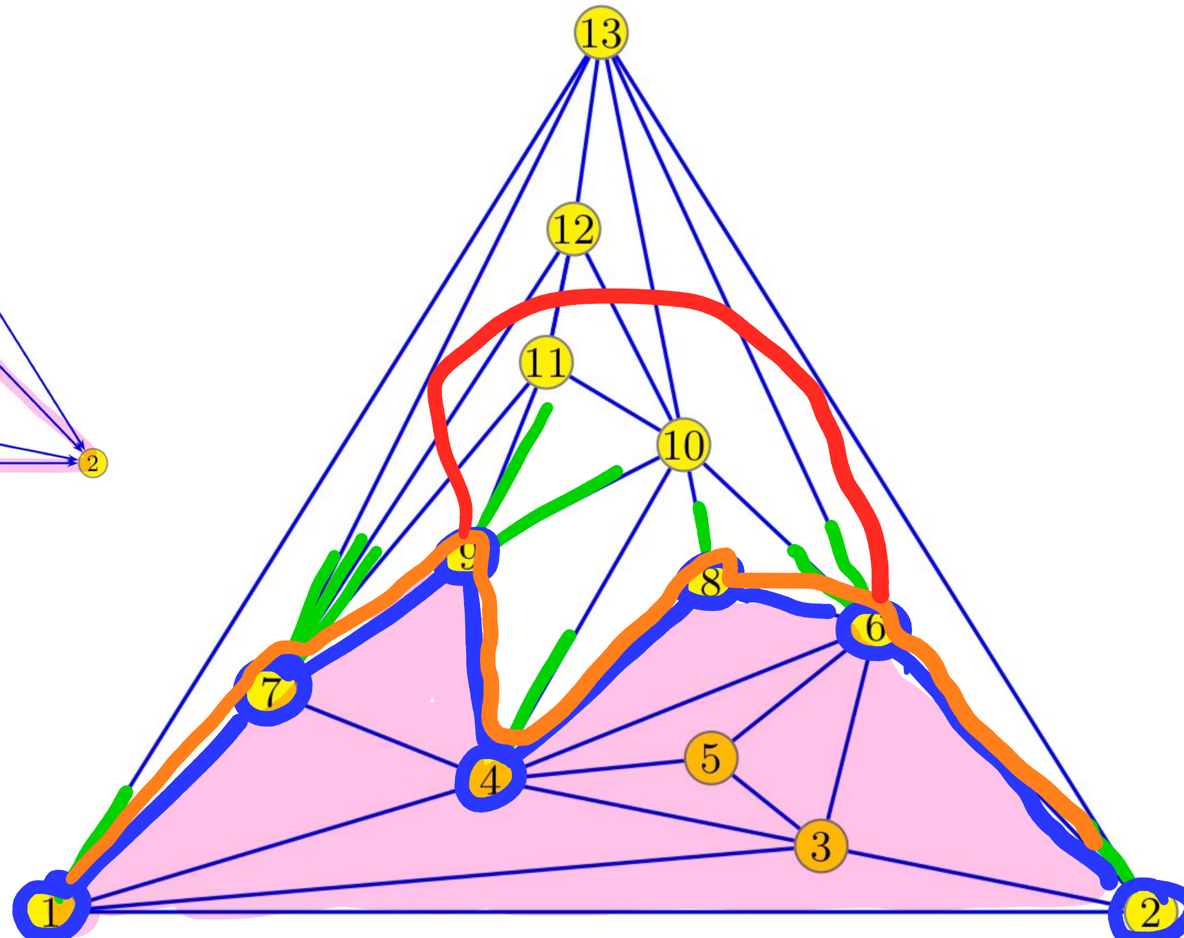


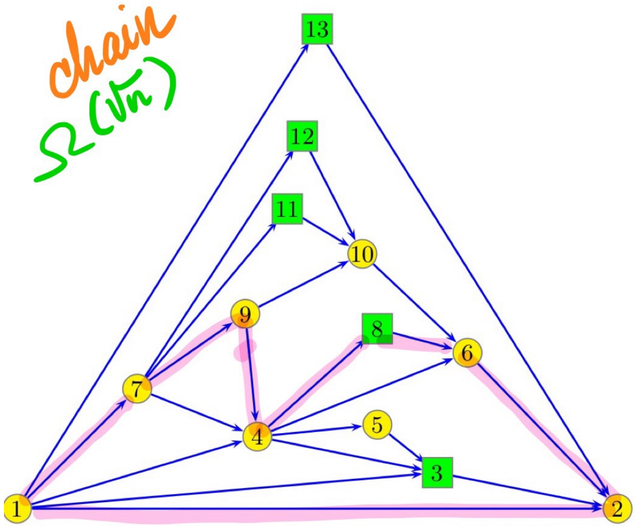


chain

$\Omega(\sqrt{n})$






chain
Ω(√n)
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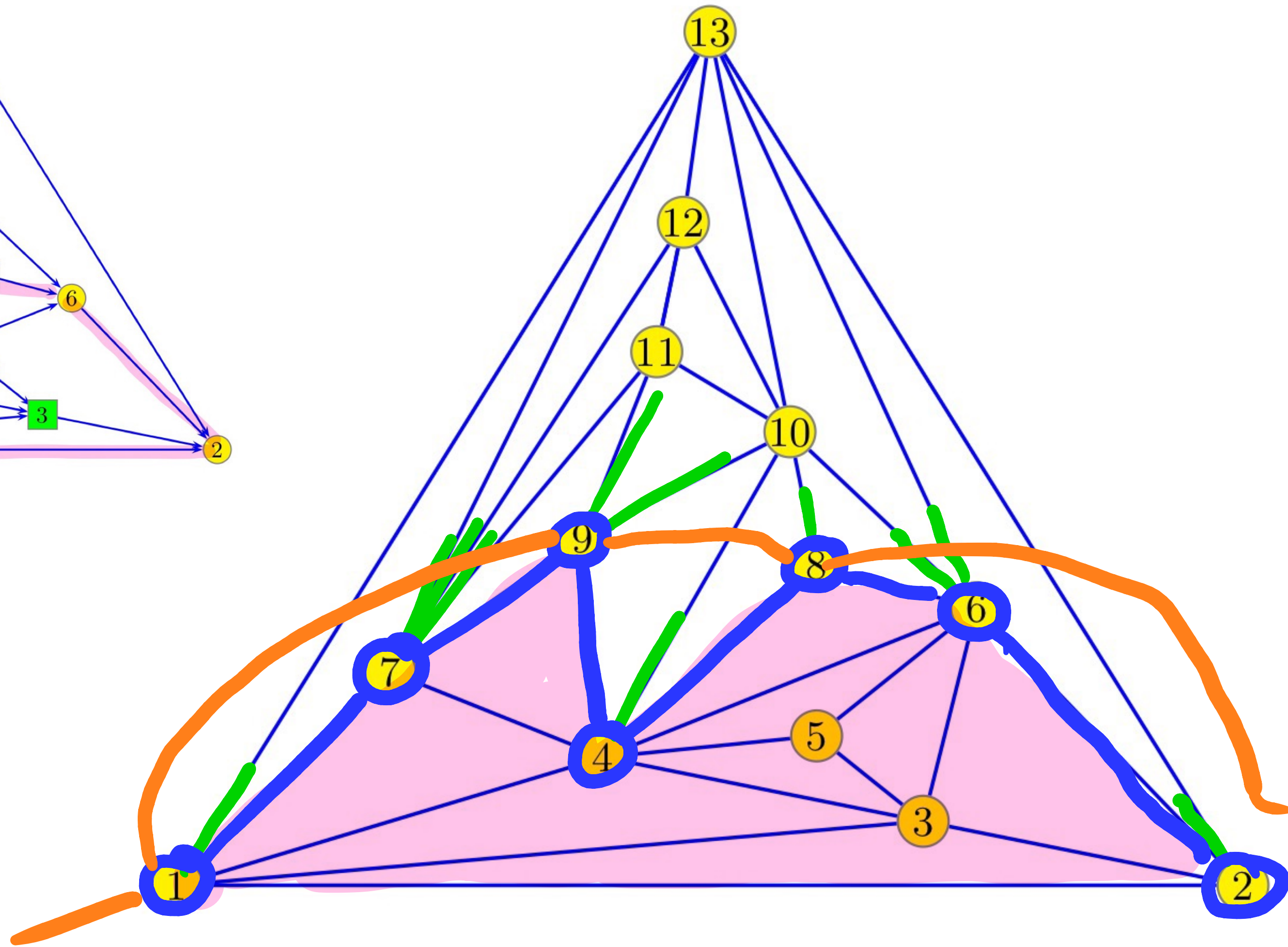
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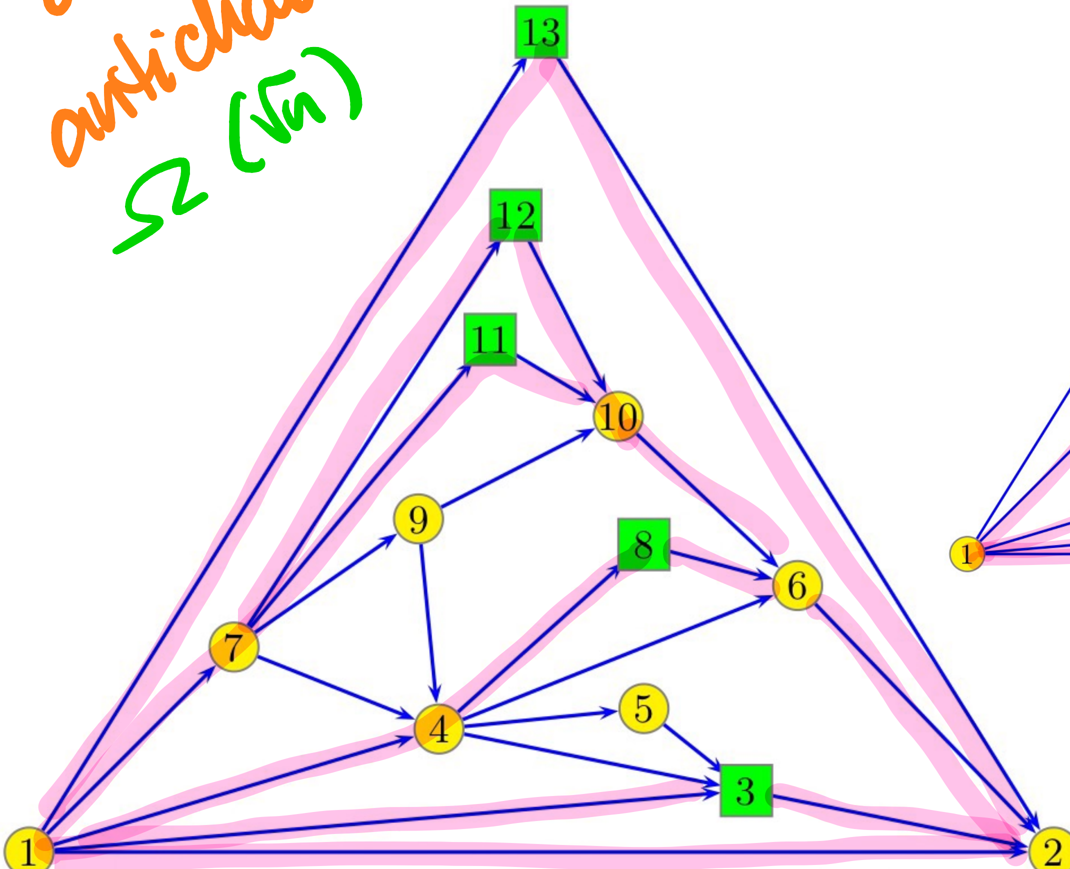
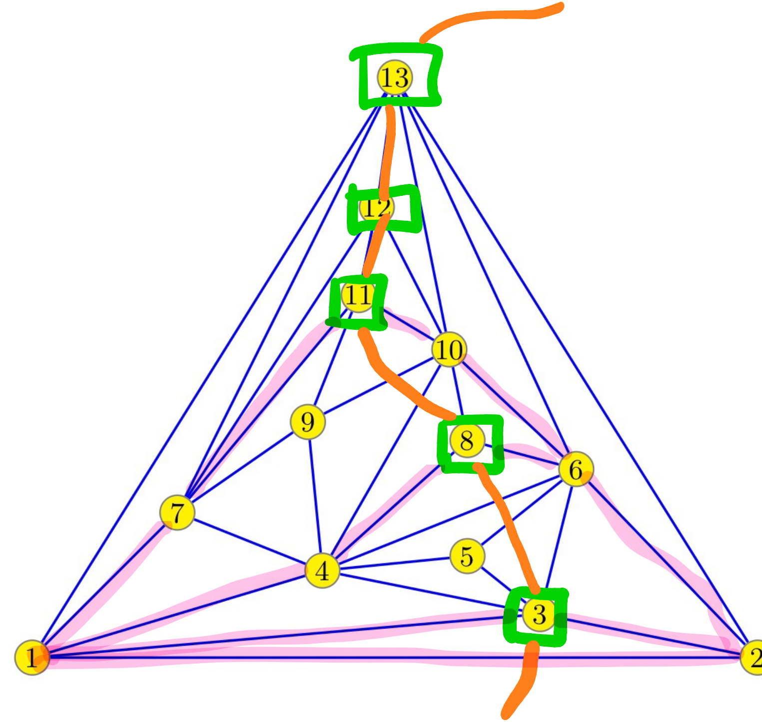
or antichain $\Omega(\sqrt{n})$

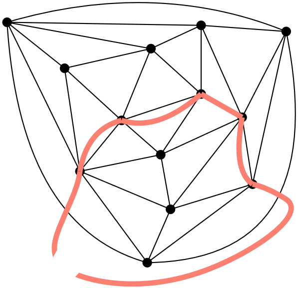




Proper Good Curves and Dual Cycles

tool past results

- Every proper-good curve containing k vertices gives a dual cycle of length at least k.



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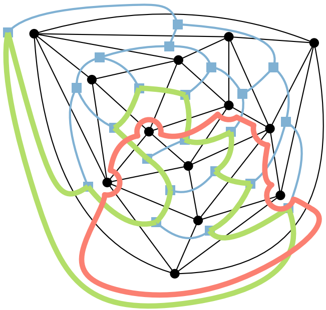

- What about the other direction?
- Can we get a proper good curve containing many vertices from a long dual cycle?

Dual Cycles—Circumference

- The longest cycle in $G^\star$ is called its *circumference*

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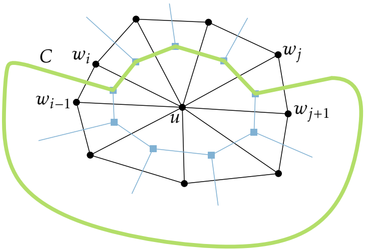
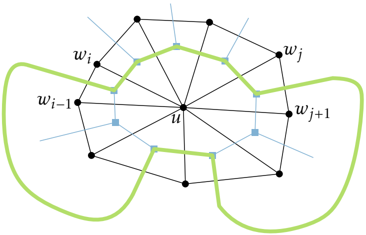
Dual Cycles and Proper Good Curves

- Every dual cycle defines a proper good curve





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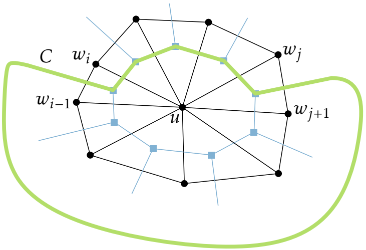

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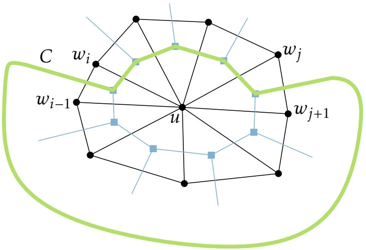

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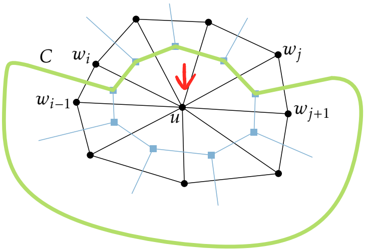

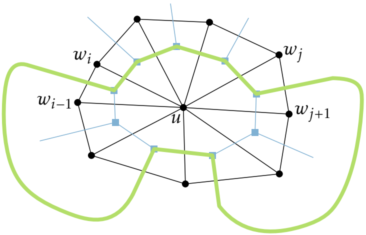

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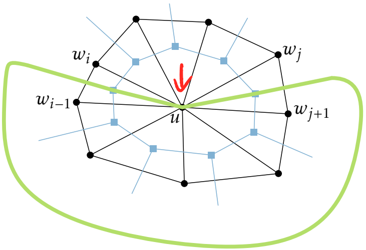

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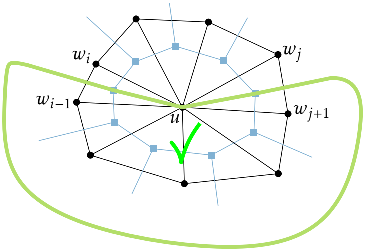

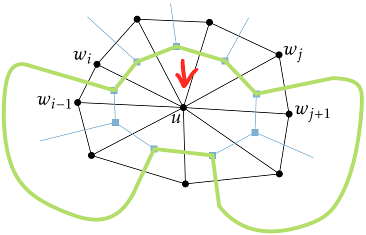

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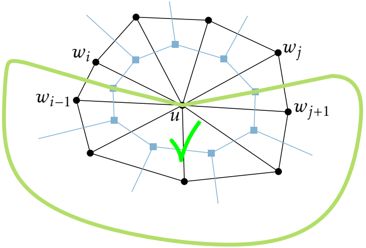

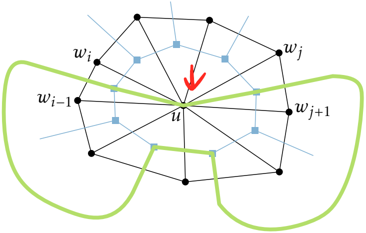

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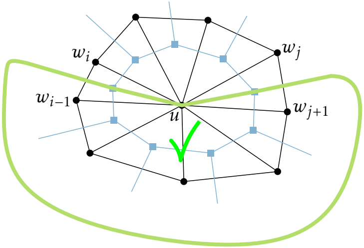

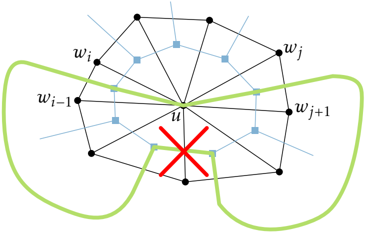

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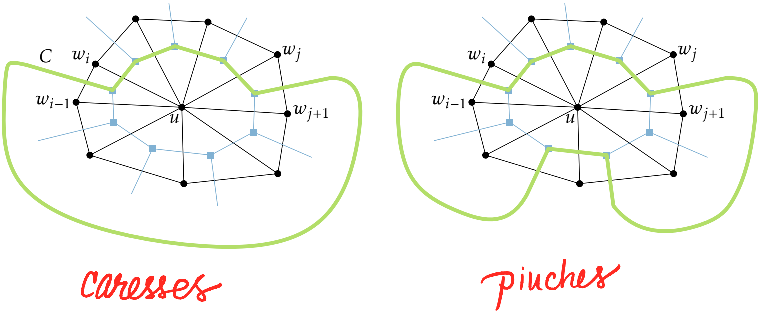
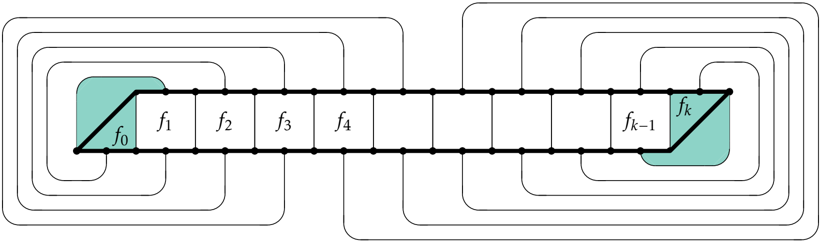


A Bad Example



Theorem: If $G^\star$ has a cycle of lenth ℓ, then $G^\star$ has a cycle C' that caresses $\Omega(\ell/\Delta^4)$ faces.

Consequence: Every n-vertex planar graph of maximum degree Δ has a free set of size $\Omega(n^{0.8}/\Delta^4)$.

Open Problem: Eliminate the dependence on Δ.

Known vs Open

Size of proper good curve = (free) collinear set

PLANAR Graph Class	Lower bound ✔	Upper bound	
outerplanar	$\frac{n}{2}$	$\Theta(n)$	2004
series-paralel	$\Omega(n)$	$\Theta(n)$	2005
3-trees	$\Omega(n)$	$\Theta(n)$	2016
cubic 3-connected	$\Omega(n)$	$\Theta(n)$	
bounded degree	$\Omega\left(\frac{n}{\Delta^4}\right)$	$\Theta(n)$	
all planar	$\Omega(n^{1/2})$	$\Theta(n^{\log_{23} 22}) \leq \Theta(n^{0.98..})$	

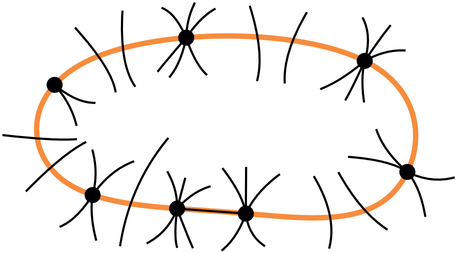
– UNIVERSAL SUB (POINT) SET

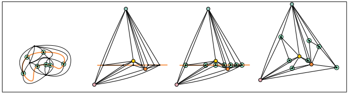
Proper Good Curves and Collinear Sets

Theorem (Da Lozzo, Dujmović, Frati, Mchedlidze, Roselli (2018))

S is a collinear set iff some proper good curve contains S.

Proof sketch:





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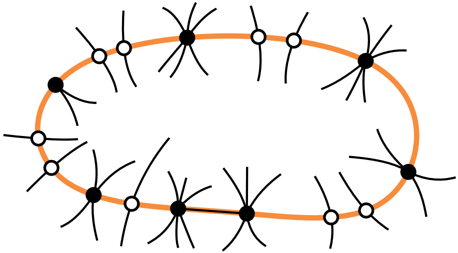

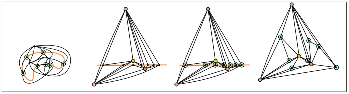

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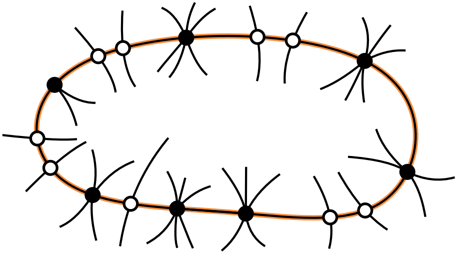

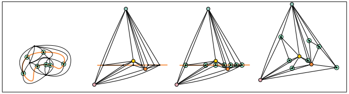

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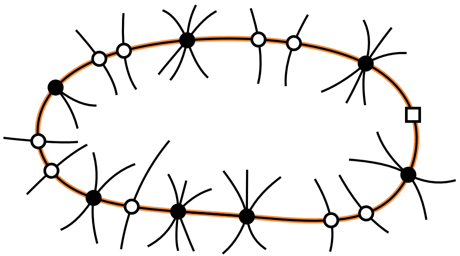

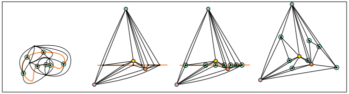

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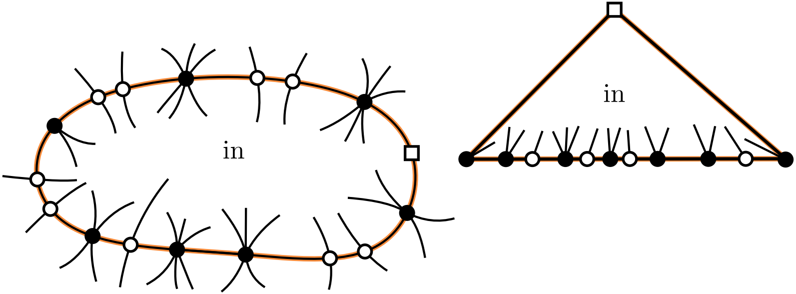

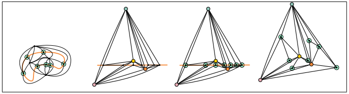

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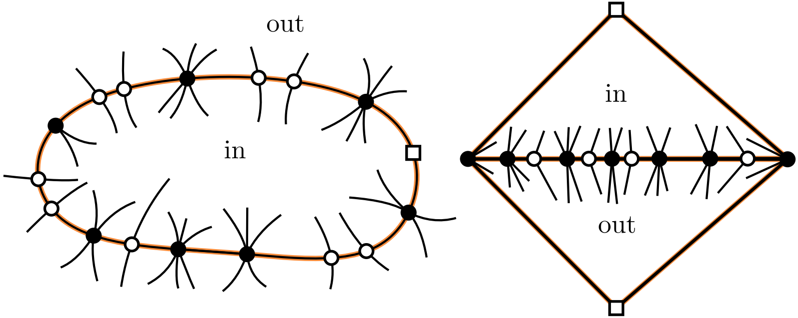

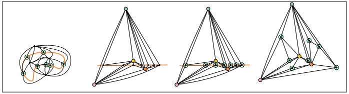

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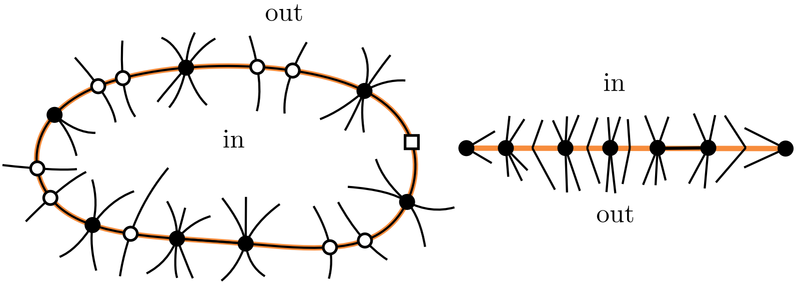

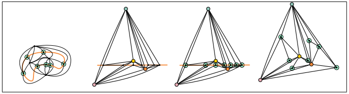

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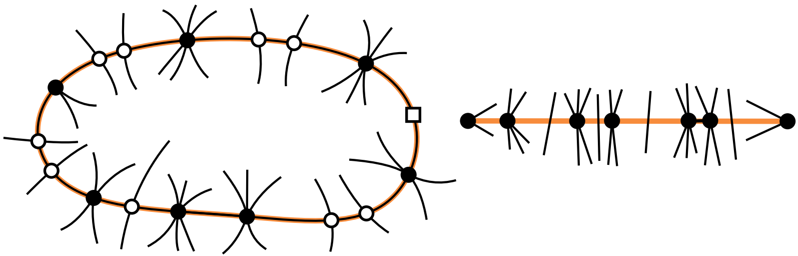
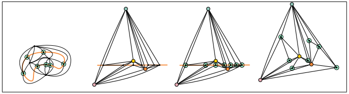
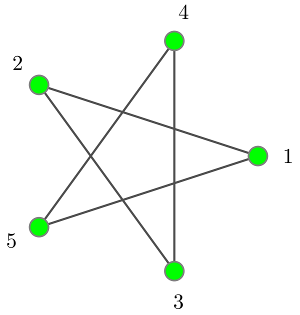
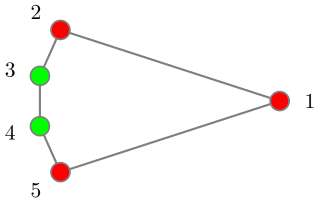




Example





fix(**G**) $\stackrel{\text{def}}{=}$ the maximum integer **t** s.t. **G** can be untangled while keeping **t** vertices fixed.

previous work: cycles

Can every cycle be untangled while keeping $\varepsilon \mathbf{n}$ vertices fixed?

[Watanabe 1998]

- $\exists$ inf. many cycles $\mathbf{G}$ with $\text{fix}(\mathbf{G}) \leq \mathbf{c}\,(\mathbf{n} \log \mathbf{n})^{2/3}$. [Pach and Tardos:2002, DCG]

planar graphs

open question:

Can planar graphs be untangled while keeping $\mathbf{n}^{\varepsilon}$ vertices fixed?

[Pach and Tardos:2002, DCG]

Theorem

For every geom planar graph $\mathbf{G}$*,*

$$\text{fix}(\mathbf{G}) \geq (\mathbf{n}/3)^{1/4}$$

[D. & Bose, Hurtado, Langerman, Morin, Wood, 2007]

previous work: subclasses of planar graphs

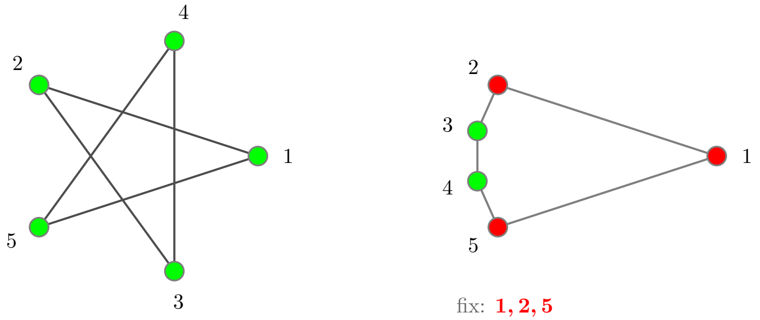
graph class $\mathcal{G}$	lower bound	upper bound
cycles	$\mathbf{\Omega(n^{2/3})}$ [Cibulka'08]	$\mathcal{O}\mathbf{(n \log n)^{2/3}}$ [Pach&Tardos'08]
trees	$\sqrt{\mathbf{n/2}}$	? [Spillner & Wolff]
outerplanar	$\mathbf{\Omega(\sqrt{n})}$ [Spillner & Wolff]	$\mathcal{O}\mathbf{(\sqrt{n})}$ [Goaoc *et al.*'07]

previous work

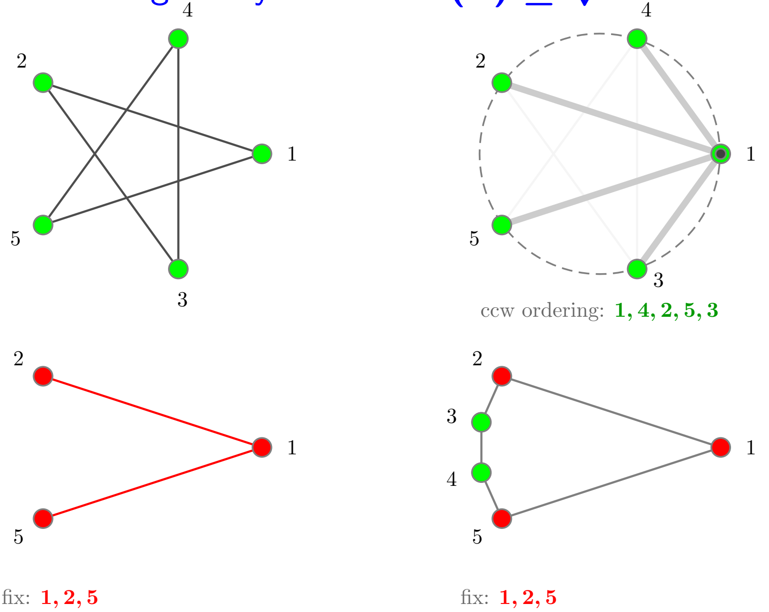
for $\mathbf{G}$ planar

- $\mathrm{fix}(\mathbf{G}) \geq \mathbf{3}$ [Goaoc *et al*, GD 2007]
- $\mathrm{fix}(\mathbf{G}) \geq \mathbf{c}\sqrt{\log \mathbf{n} / \log \log \mathbf{n}}$ [Spillner and Wolff, 2007]

Example



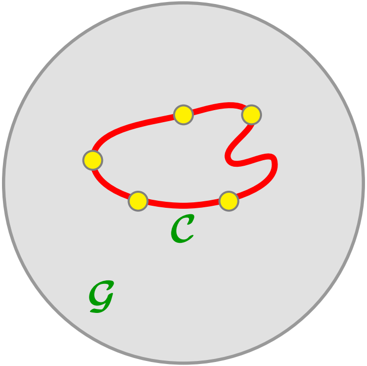
how to untangle a cycle? $\text{fix}(\mathbf{G}) \geq \sqrt{\mathbf{n}}$



By Erdős-Szekeres Theorem, #fix vertices of geom cycles is at least $\sqrt{\mathbf{n}}$.

general planar graphs: (wrong) idea

assume *triangulations*



❶ $\mathbf{C} \overset{\text{def}}{=}$ largest induced cycle in $\mathbf{G}$.

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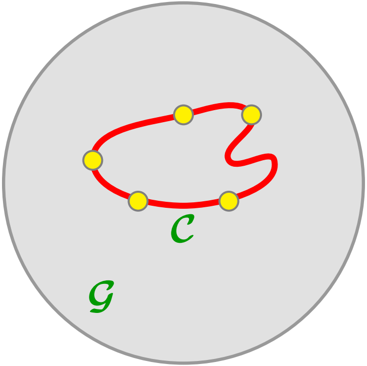

1. **C** $\stackrel{\text{def}}{=}$ largest induced cycle in **G**.
2. untangle **C** while keeping $|\mathbf{C}|^{1/2}$ vertices fixed.

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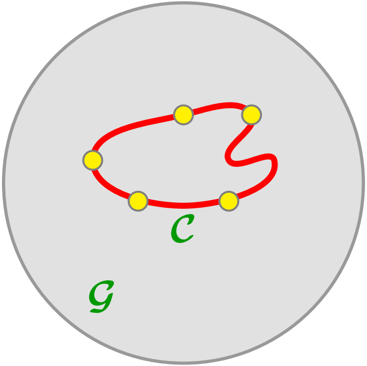
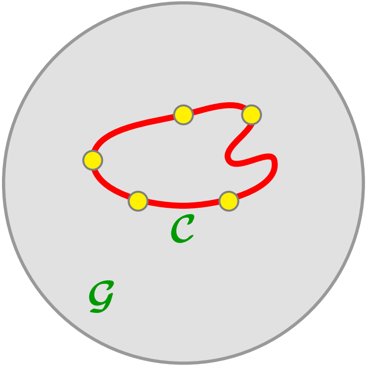


1. **C** $\overset{\text{def}}{=}$ largest induced cycle in **G**.
2. untangle **C** while keeping $|\mathbf{C}|^{1/2}$ vertices fixed.
3. move the rest of the vertices of **G**. inside of **C** in the untangling.

general planar graphs: (wrong) idea

assume *triangulations*



❶ **C** $\stackrel{\text{def}}{=}$ largest induced cycle in **G**.

❷ untangle **C** while keeping $|\mathbf{C}|^{1/2}$ vertices fixed.

❸ move the rest of the vertices of **G**. inside of **C** in the untangling.

if **C** is convex $\Longrightarrow$ $\mathrm{fix}(\mathbf{G}) \geq \sqrt{\log |\mathbf{C}|}$

two problems

(a) $|\mathbf{C}|$ may be of constant size.

(b) convexity of $\mathbf{C}$ in the untangling $\Longrightarrow$ **log** best possible

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fix for (b)

each face in the untangling of **H** is star-shaped

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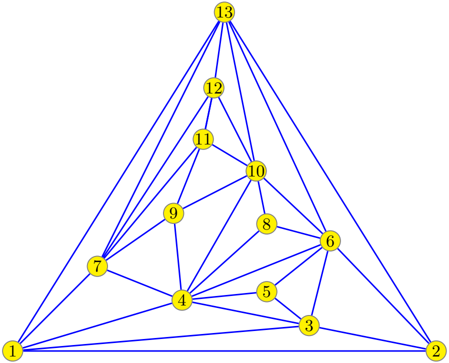
Example:
H an embedded outerplanar subgraph of $\mathcal{G}$.

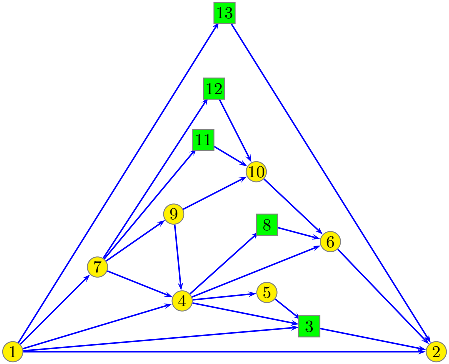
By Moore's bound, $|\mathbf{H}| \geq \lfloor \log n / \log\log n \rfloor \Longrightarrow$
$\text{fix}(\mathbf{G}) \geq \mathbf{q}\sqrt{\log n / \log\log n}$

[Spillner and Wolff]

Frame



canonical ordering of $\mathcal{G}$



frame $\mathcal{F}$

directed path in $\mathcal{F}$

$\mathbf{H} \stackrel{\text{def}}{=}$ a subgraph of $\mathbf{G}$ induced by a directed path in $\mathcal{F}$.

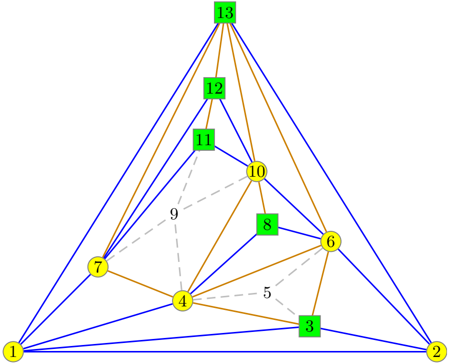
$\mathbf{H}$ is then an embedded outerplanar in $\mathcal{G}$.

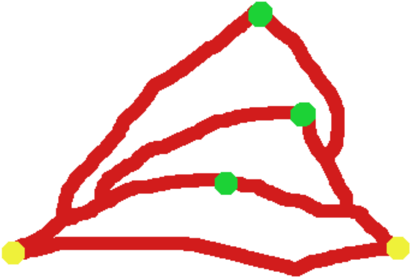
Lemma

A subgraph $\mathbf{H}$ induced by a directed path in $\mathcal{F}$ can be untangled wh ile keeping $\mathbf{c}\sqrt{|\mathbf{H}|}$ vertices fixed.

two compatible subgraphs

chain in $<_{\mathcal{F}}$ ⇒ induces an embedded *outerplanar* subgraph **H** in $\mathcal{G}$
antichain in $<_{\mathcal{F}}$ ⇒ ??? subgraph.





putting it all together

By Dilworth's Theorem:
$<_{\mathcal{F}}$ contains a chain of size $\sqrt{\mathbf{n}}$, or an anti-chain of size $\sqrt{\mathbf{n}}$.

geometric lemmas:

Lemma
A chain $\mathbf{H}$ in $<_{\mathcal{F}}$ can be untangled while keeping $\mathbf{c}\sqrt{|\mathbf{H}|}$ vertices fixed.

Lemma
An antichain $\mathbf{H}$ can be untangled while keeping $\sqrt{|\mathbf{H}|}$ vertices fixed.

summary and open problems

graph class $\mathcal{G}$	lower bound	upper bound
planar	? [Pach&Tardos'02]	$\mathcal{O}(\sqrt{n})$ [Goaoc *et al.*'07]

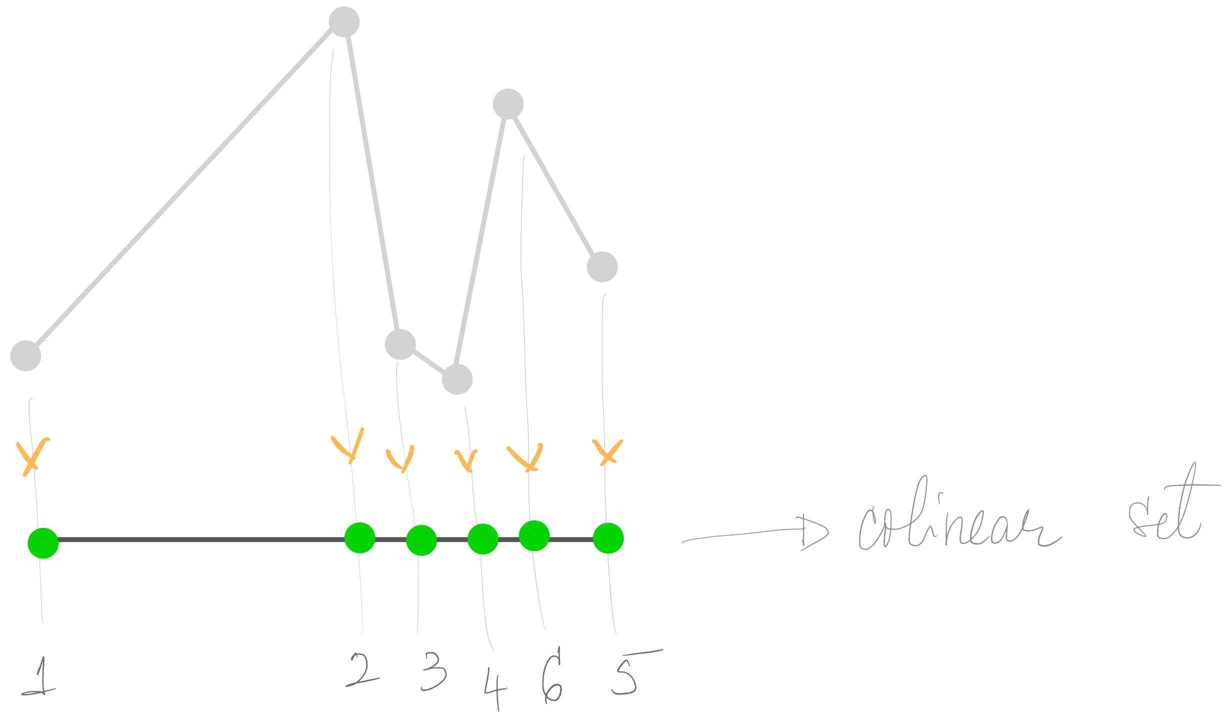
summary and open problems

graph class $\mathcal{G}$	lower bound	upper bound
planar	$\mathbf{\Omega(n^{1/4})}$	$\mathcal{O}(\sqrt{\mathbf{n}})$ [Goaoc *et al.*'07]

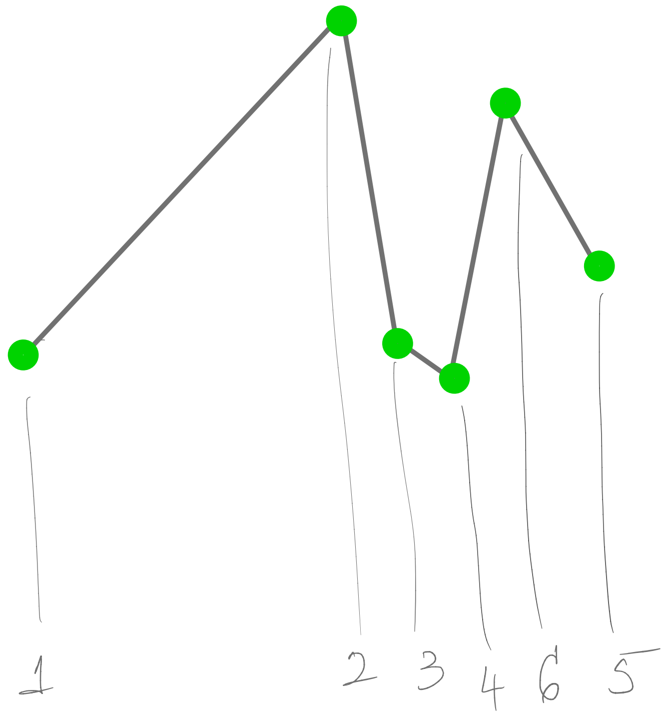
open problem: Close the gap for planar graphs?

graph class $\mathcal{G}$	lower bound	upper bound
cycles	$\mathbf{\Omega(n^{2/3})}$ [Cibulka'08]	$\mathcal{O}(\mathbf{n \log n})^{2/3}$ [Pach&Tardos'02]
trees	$\sqrt{\mathbf{n/2}}$	? [Spillner & Wolff'07]
outerplanar	$\mathbf{\Omega(\sqrt{n})}$ [Spillner & Wolff]	$\mathcal{O}(\sqrt{\mathbf{n}})$ [Goaoc *et al.*'07]

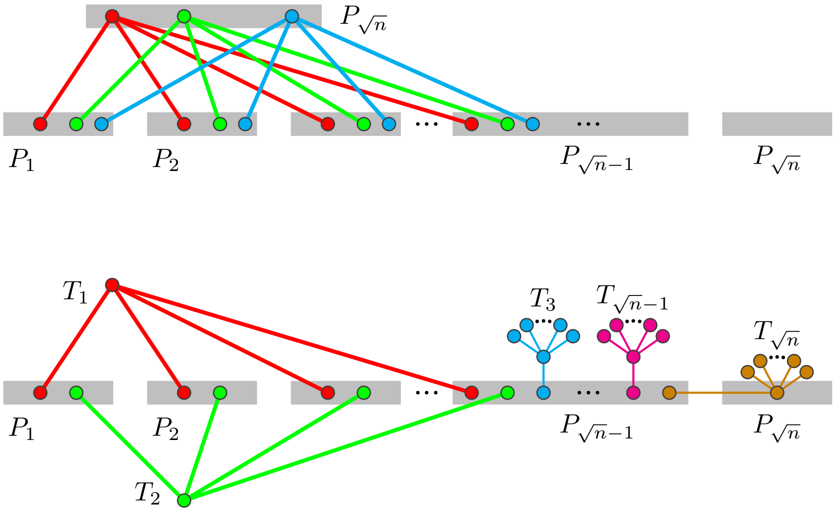
Collinear Sets



Collinear Sets



trees-upper bound: fix(T) = $3\sqrt{n} - 3$



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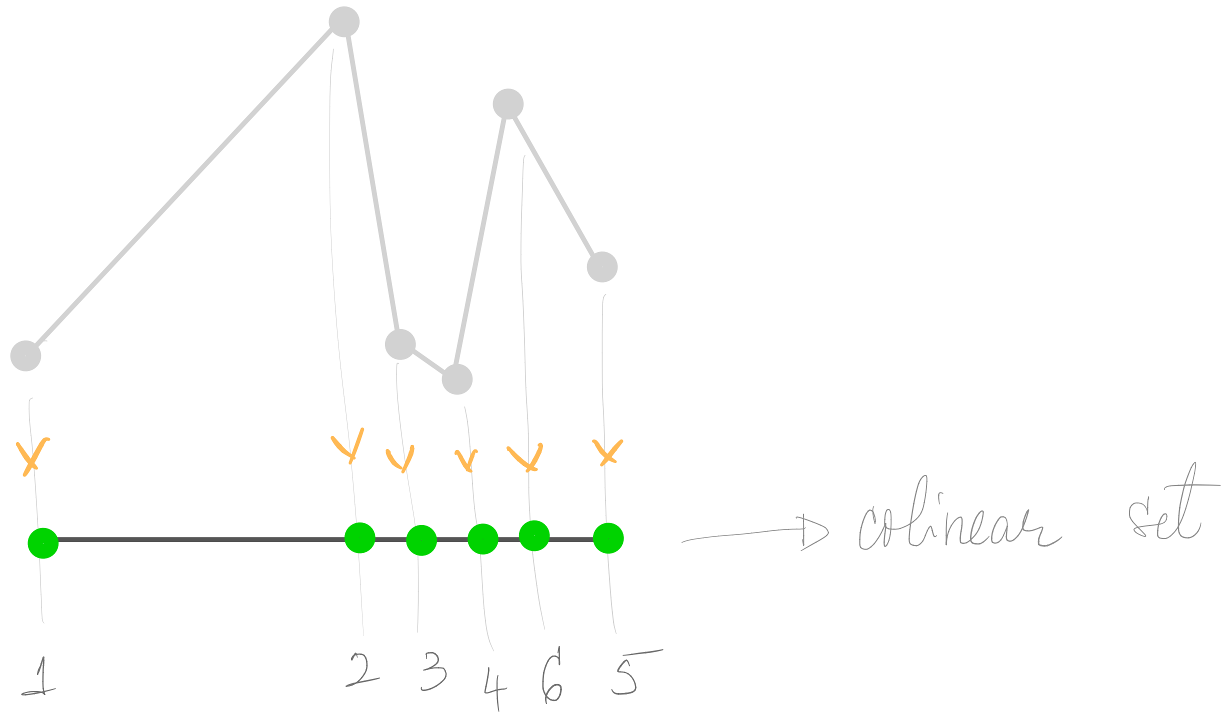

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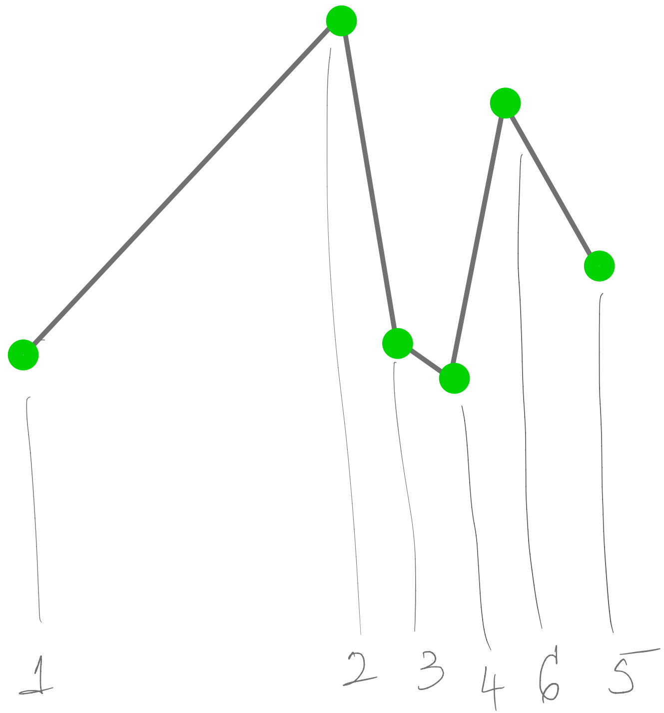

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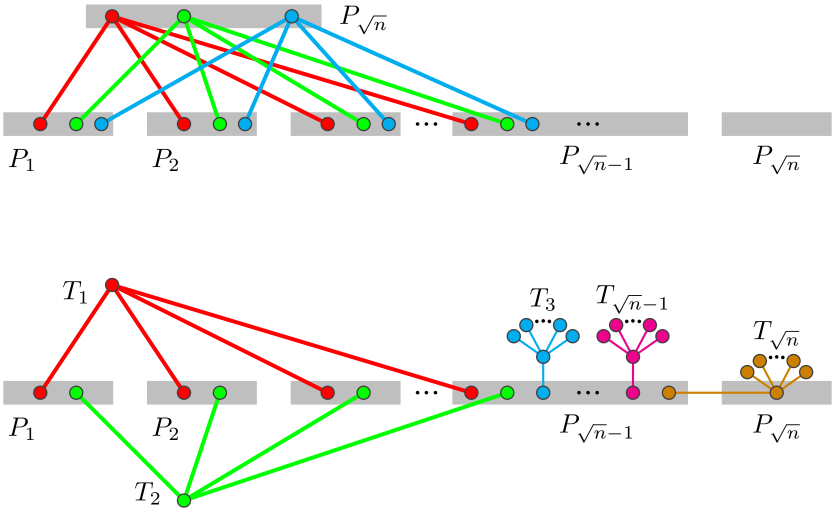

Why Collinear Sets?